ERROR RATE ANALYSIS OF INTERFERENCE SPREADING IN COGNITIVE RADIO NETWORKS

By

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As wireless technology continues making giant leaps on a daily basis, the current legacy spectrum allocation system will not hold good for long. Upcoming technologies like Artificial Intelligence and Internet of Things demand massive bandwidths and super-high data rates. There is a looming spectrum shortage crisis which could lead to the demise of wireless communications. Cognitive Radio (CR) with its smart and efficient use of radio spectrum is proposed as a solution to this crisis. It exploits the spectrum holes generated by the legacy allocation system, to transmit information of unlicensed users. The main function of a CR system, is to share the available spectrum efficiently between a primary user (PU) and a secondary user (SU). PU's are given better quality of service (QoS) while SU's can use the spectrum with co-operation from the PU. Spectrum sensing is the core of a CR system as it senses whether the spectrum is occupied by a PU. If not in use, the CR system permits a PU to transmit information through the same frequency band. In this way, a CR can efficiently share spectrum, which unclogs the available spectrum. However, the main issue with spectrum sensing is its unreliability in dynamic environments. It is also extremely power hungry since it has to monitor the spectrum at all times for idle bands. This makes spectrum sensing impractical for power-limited systems. Thus, in our work, we use a method called "interference spreading" to distribute interference, along with "interference temperature". Interference spreading is a viable alternative to the power-hungry spectrum sensing. In our work, absence of spectrum sensing is assumed, which would lead to collisions between PU's and SU's. Orthogonal Frequency Division Multiplexing (OFDM) method is assumed, where sub-carriers are assumed to be allocated randomly. We then use Bit Error rate (BER) as a metric to determine the performance of the system. The SU's power is varied while PU's power is kept constant, along with a fixed value for interference temperature. BER of a SU is calculated for various modulation schemes. The main contribution of this thesis is the performance analysis of a CR system with the use of interference spreading method.

Keywords: Spectrum, Cognitive Radio, Spectrum Sharing, Interference Temperature, Interference Spreading, Bit Error Rate, Modulation Schemes, Sub-carrier Collisions.

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INTRODUCTION

1.1 Current Allocation Scenario of Radio Spectrum

In todays digital age, advances in communication technology has led to the proliferation of digital devices which communicate wirelessly like phones, computers, tablets etc. It has been well documented that any kind of electronic communication between two devices requires a transmission medium or communication channel to carry the information from a source to the destination. For wireless or radio communication, the transmission medium is free space or the air around us. The wireless or electromagnetic waves from the source (transmitter) travel at the speed of light to reach the destination (receiver) through free space and this means that these electromagnetic waves are all around us. However, this system of transmitting electromagnetic waves through the air would be impractical as there are millions, or even billions of devices trying to communicate with each other. The number of devices competing for communication would increases the probability of interference between the electromagnetic waves. Interference would lead to a loss of information and eventually, a total failure in communication [1].

The simple solution to the interference problem was that each communication is assigned its own frequency and the transmitters and receivers would operate in only their allotted electromagnetic frequencies [2]. This mitigated the problem of interference to a large extent as each communication had its own fixed frequency to operate in and this solution also improved the quality of communication. The government of each country has the sole responsibility to manage and allocate the spectrum available at its disposal. In the United States, the Federal communications commission (FCC) manages all commercial and domestic frequencies while another government agency called the National Telecommunications and Information Administration (NTIA) handles government and military frequency assignments. The frequency spectrum assigned universally for wireless or radio communications is from roughly 30 kHz to 300 GHz [3]. This allocation of distinct frequencies to different communications seemed ideal until the recently. However, due to a rapid evolution of technology the number of wireless users has increased exponentially. This increase in the number of users has left the electromagnetic spectrum tremendously crowded, as can be seen from the Figure 1.1. As the number of devices only increase on a daily basis, some alternative has to be found to solve the diminishing spectrum crisis or soon there will be no frequency bands available for wireless communication.

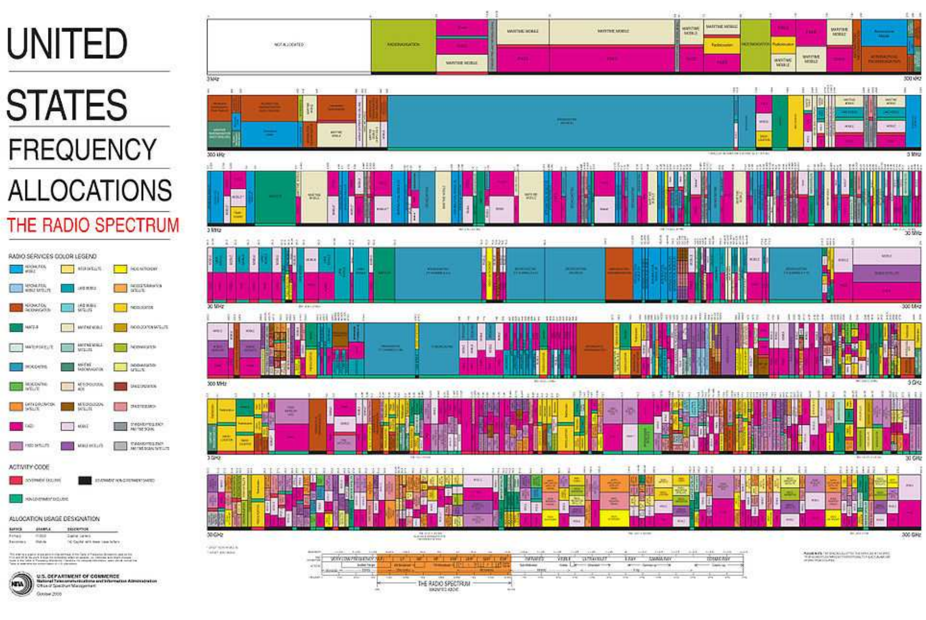

Figure 1.1: NTIA spectrum allocation chart [4].

Figure 1.1 clearly shows the overcrowding of the electromagnetic spectrum. Most of the frequencies from 30 kHz to 300 GHz have already been allocated for various communications. This spectrum allocation chart was provided by the NTIA in October 2003 and the issue of spectrum crowding has only steadily increased since then due to the inception of new technologies like smart devices, Internet of things, wearables, etc. New technologies today also demand faster data rates. Faster data rates can be achieved if we increase the allocated frequency spectrum for a communication. But, this again leads to the concern of spectrum shortage. In addition, surveys conducted have suggested that the radio spectrum is being used in an extremely inefficient manner and has been over-exploited [5, 6, 7, 8]. This is shown in Figure 1.2.

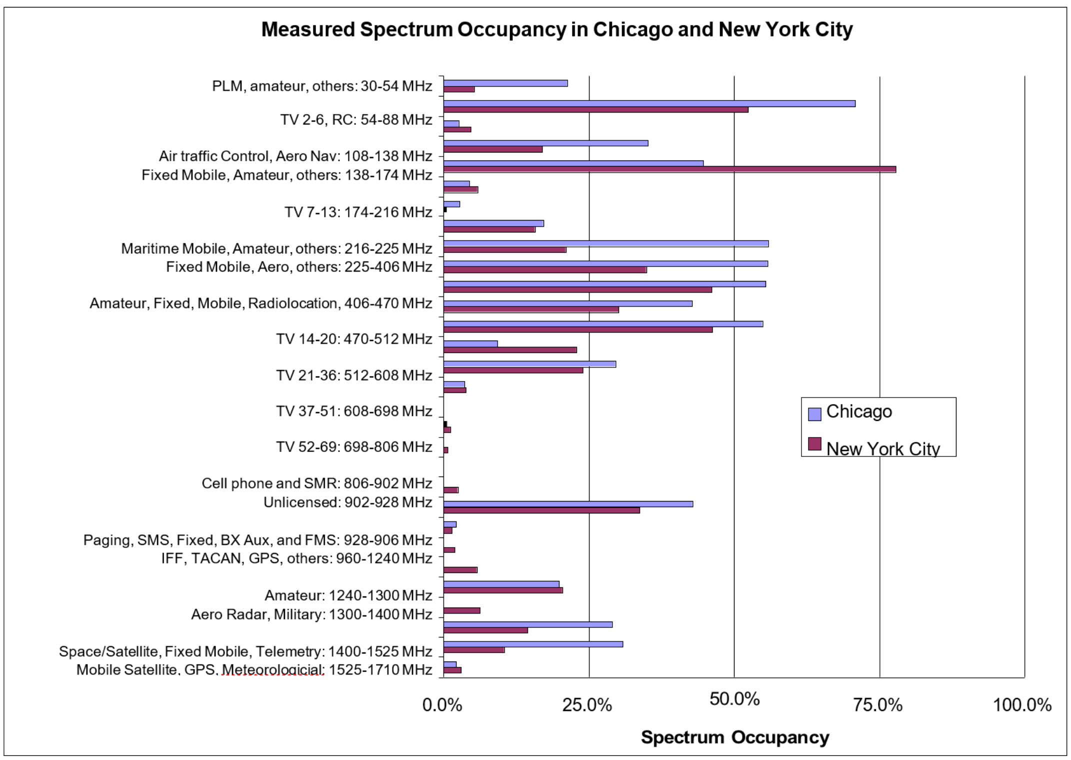

Figure 1.2: Spectrum occupancy in Chicago and New York City [5].

Figure 1.2 shows that the utilization of certain bands is over 50% where some bands are not even being utilized up to 5%. This demonstrates the uneven and under-utilization of the frequency spectrum which remains a concern. We need to utilize the allocated spectrum with more care and efficiency. In addition, Cisco predicts that by the year 2021, data produced by smart phones would increase tremendously as shown in Figure 1.3, which would lead to unimaginable spectrum demands [7].

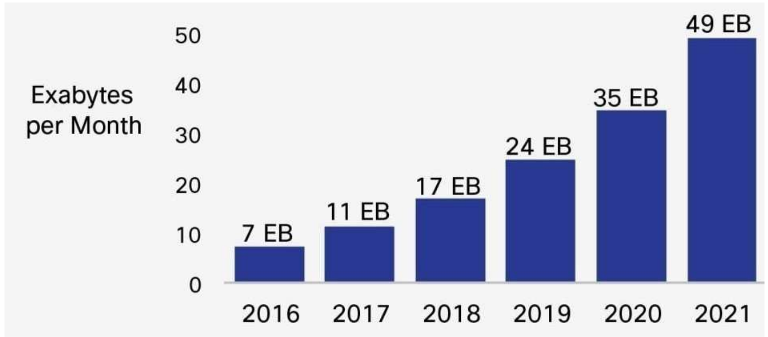

Figure 1.3: Estimated growth in data usage by smart-phones [7].

To summarize, the limited spectrum we have is fast diminishing while the amount of wireless users and devices grow. In addition, the amount of data and the need for faster data rates is rising rapidly. It is clear that we spectrum is not utilized efficiently. If current trends are followed, the future of wireless communication looks bleak. Steps have to be taken quickly to solve the crisis.

1.2 Potential of Cognitive Radio (CR)

As we have already established, the demand for wireless bandwidth or spectrum is ever increasing and the traditional system of allocating fixed frequencies for each communication will not hold good for long. The spectrum is scare and no one knows how much bandwidth future technologies would require. So, we need a solution that is dynamic and can adapt quickly to varying bandwidth requirements. Also, we need to able to integrate this solution easily with the current system or risk facing back-lash from the existing users. Taking all these parameters into consideration, a novel concept called ”Cognitive Radio (CR)” was proposed [9][10]. It was first proposed by Joseph Mitola III, in a seminar at the KTH (the Royal Institute of Technology at Stockholm) in 1998 along with Gerald Q. Maguire Jr. It is a technology which takes advantages of the unused frequencies or spectrum holes created by the allocated or fixed frequencies [11]. CR utilizes these spectrum holes to transmit data thereby increasing efficiency of the spectrum utilization. The Federal Communications Commission (FCC) defines CR as "a radio that can change its transmitter parameters based on interaction with the environment in which it operates [12]." and Simon Haykin defines it in his pioneering paper as : "The CR, built on a Software-Defined Radio (SDR), is defined as an intelligent wireless communication system that is aware of its environment and uses the methodology of understanding-by-building to learn from the environment and adapt to statistical variations in the input stimuli" [13]. This means that the CR is an intelligent system which is aware of its electromagnetic environment (which frequencies are currently being used and which frequencies are not) and it uses this information to dynamically assign frequency bands for communications in the most optimum way possible without collisions. The primary objectives of CR could be stated as follows:

1. Highly reliable communication whenever and wherever needed [13].

2. Efficient use of the radio spectrum [13].

In Figure 1.4, radio environment is the electromagnetic spectrum. The CR is a smart wireless communication system which is aware of the electromagnetic spectrum occupancy status. The CR constantly observes or senses the radio environment for spectrum holes. It then analyses the spectrum holes and decides which spectrum hole or frequency band would be optimal for transmission. It has to keep in mind that this allocation should not cause any interference to the other communications. Simultaneously, the system is constantly learning and observing the patterns so that better decisions can be taken in future about optimum allocation. The last step is to orient or adapt to the current spectrum scenario by changing parameters like transmit power, frequency etc.. This will ensure the most efficient use of the radio spectrum. Such a system has the potential to solve the spectrum crisis by exploiting

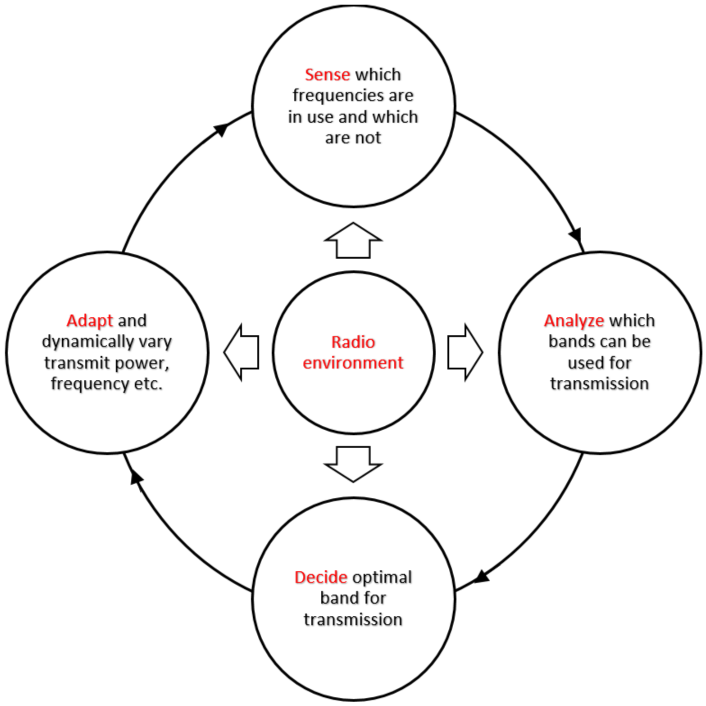

Figure 1.4: Block diagram of a Cognitive Radio system.

the spectrum holes.

1.3 Applications of Cognitive Radio

CR has numerous applications such as:

1. Cellular mobile networks: The most obvious application of cognitive radio would be in cellular networks because of the increase in demand for higher data rates [14, 15].

2. Public safety communications: During times of natural calamities, communication lines can break down and isolate certain areas, endangering lives of many people. In such times of emergency, CR based networks can form mesh networks and tune to a non congested or operational band for reliable and quick communications.

3. Smart grids: Smart power grids require multi-way communication between different entities, ranging from power generators to transmitters and users. CR, being a smart network can find applications in this area.

4. Vehicular networks: CR can be used in vehicle to vehicle communications which is considered the future of the automobile industry. This application is being researched extensively [16].

5. Communications in outer space: CR is being considered as the next big means of inter-satellite communications. Smart and multi-way communications is required for inter satellite communications and this is exactly what CR can offer [17].

1.4 Limitations of Cognitive Radio

Even though CR has the potential to be the solution for all the spectrum related issues, it is only in its development stages and has never been implemented on a large scale. Following are some of the concerns related with CR:

1. The major concern is the wrong detection of a free band or as it is called hidden primary user problem. If CR mistakes an occupied band for a free band and goes ahead with information transfer from another transmitter, information from both parties could be lost and communication would break down [18].

2. The learning stage has to be constantly supervised as one wrong step could mean that the entire knowledge of the CR system will be effected. High level machine learning techniques have to be applied here [19].

3. Security issues are a major problem with CR systems. We know that enforcement of security guidelines for static systems is already a huge challenge. The

amount of resources required to authorize equipment, the requirement of obtaining proof that violations have occurred are big challenges. As systems progress and become more dynamic, like in CR, the number of potential interactions can only lead to an increase in violations causing concerns over security [19].

4. Implementation challenges are also not well researched. Even though CR is booming and interesting results have confirmed its legitimacy, doubts still exist over its physical implementation and infrastructural needs [20].

5. The optimum modulation schemes are also being looked into while there are no existing standards yet for CR. The above challenges confirm there are still some huge hurdles to be crossed for a full scale implementation of CR in the real world. However, it is only a matter of time before these issues are resolved and CR is well in line to be the future of efficient and reliable spectrum access.

1.5 Organization of Thesis

In Chapter 2, a detailed background on CR is presented. Parameters defining CR functioning will be described. In Chapter 3, Bit Error Rate (BER) is first defined, then the modulation schemes used in this thesis will be reviewed. Chapter 4 presents a detailed analysis of the results obtained in this thesis. Chapter 5 includes future work along with the conclusions.

CHAPTER 2

COGNITIVE RADIO

In this chapter, we are going take a detailed look at cognitive radio operations with the help of the CR cycle, and also understand the importance of spectrum sensing and interference temperature (IT).

2.1 Cognitive Radio

Cognitive radio (CR) is an intelligent radio system which is aware of its electromagnetic environment (which frequency bands are currently in use and which frequencies are not). A CR uses this information to dynamically assign frequency bands to other users who do not have an assigned frequency for transmission. Hence, CR efficiently uses the spectrum. Its main aim is to provide highly reliable communications while using the spectrum available in the most efficient manner. Important CR terminology:

- Primary user (PU): A PU is someone who has obtained regulatory permissions or license to operate in a particular frequency band. PUs are referred to as a licensed user [21].

- Secondary user (SU): A SU is someone who does not have the license to operate in a particular frequency band but, can use the frequency band with the co-operation of the PU referred to as an unlicensed user [21].

- Spectrum sharing: A CR has the ability to intelligently adapt to its wireless environment by changing certain parameters such as transmit power. The main function of this, is to dynamically share the spectrum between primary and SUs without any collisions. This greatly improves spectrum efficiency [22].

- Spectrum sensing: One of the main functions of a CR is to sense whether a particular frequency band is available for transmission or, is already in use by a PU. If a frequency band is not in use by any PU, a SU in need of frequency spectrum will be allocated the idle spectrum. [23].

- Spectrum hole: A spectrum hole is created when a particular frequency band is not in use currently. A CR takes advantage of such a spectrum hole and allows some other user to transmit information through that frequency band. The generation of spectrum holes is one of the main shortcomings of the legacy spectrum allocation system. CR solves the problem of spectrum holes and improves the spectrum efficiency exponentially. It can be better understood from the below figure.

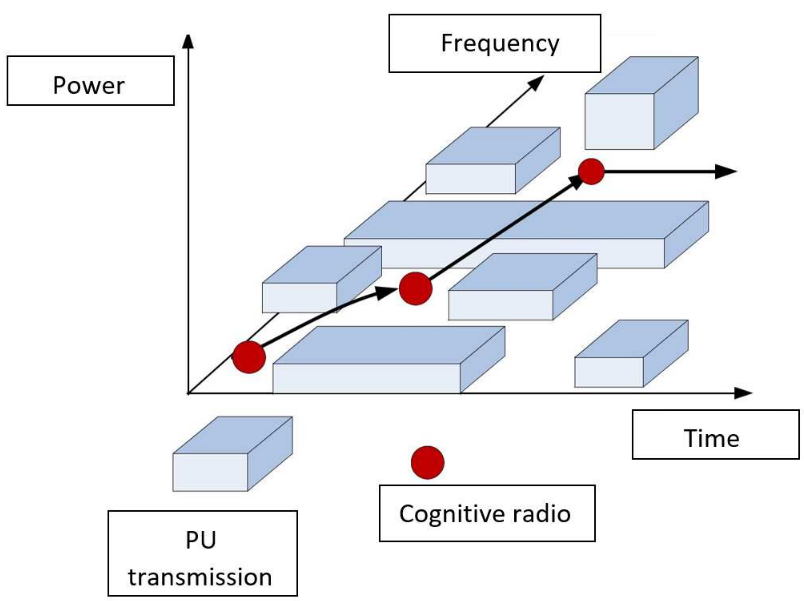

Figure 2.1: Generation of spectrum holes [21].

Figure 2.1 illustrates how a CR takes advantage of the spectrum holes. The spectrum holes are generated by the unused frequency bands of a PU. A CR senses this unoccupied spectrum and transmits information of some other user on the same frequency band, hence solving the problem of unused bands.

2.2 The Cognitive Cycle

A cognitive cycle illustrates the working of a CR. It was first defined by Mitola in 2001 [24], which was built upon and improved by Haykin later on in 2005 [13].

2.2.1 Mitola's Cognitive Cycle

Mitola first introduced the concept of CR and defined an elaborate architecture for CR, with 6 main functions or features. The external world is the radio environment in which the CR is being deployed in. The external world provides stimuli or input for the CR to observe or sense.

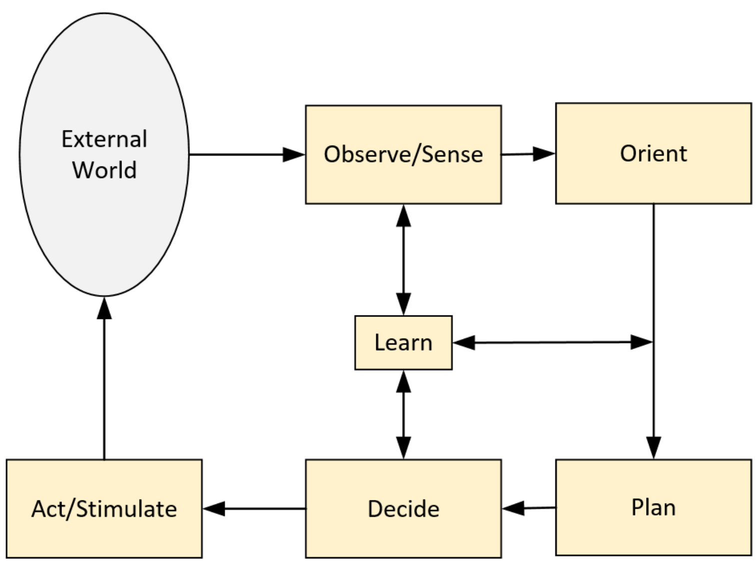

Figure 2.2: Mitola's cognitive cycle [22].

The radio environment is observed by the CR to understand the context of its communication tasks. For example, if a military communication is taking place via the environment and simultaneously, a recreational phone call is taking place, the radio has to be able to decide the urgency of each communication. The observe stage is a primary and very important stage for CR. It understands what type of communications are taking place in its environment. The next stage is the orient stage. The orient stage has to decide on the urgency or priority of each communication, based on the cues obtained from the observe stage. The plan stage, normally generates viable alternatives. The decide stage decides which alternative would be the most efficient one while allocating radio resources and sending out commands to the CR software. The act stage initiates tasks with the specified resources. The learn stage is basically, machine learning, where the cognitive radio keeps a database of all the prior information about the radio environment, information context and optimum decision routes. The dilemma however, is to whether supervise this learn stage or not. Supervision, requires a lot of overhead and power but not supervising it may cause issues. The CR could misinterpret information or learn wrongly, and when this misinterpreted decisions are used in future, could hinder communications. Finally, Mitola summarizes CR as Cognitive radio is a goal-driven framework in which the radio autonomously observes the radio environment, infers context, assesses alternatives, generates plans, supervises media services and learns from its mistakes [24, 13].

2.2.2 Haykin's Cognitive Cycle

Haykin's cognitive cycle, in comparison to Mitola's, is very compact and well structured. It consists of 3 cognitive tasks, namely radio scene analysis, channel identification and dynamic spectrum management. Haykin, also stressed upon the importance of re-configurability for CR systems, which is provided by a software defined radio (SDR) platform. SDR is basically a confluence of the two major technologies namely: digital radio and computer software [13, 25]. Re-configurability forms the basis for the following features:

- Adaptability to new and varying radio interface standards.

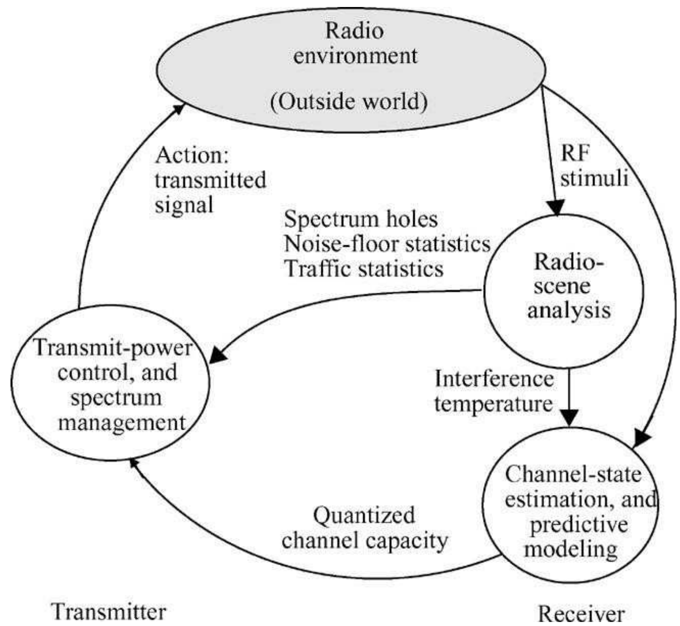

Figure 2.3: Haykin's cognitive cycle [13].

- Ability to accommodate new technologies and services.

- Incorporation of updates in software technology.

Re-configurability is not a cognitive task as it depends on the SDR. For cognitive tasks, the CR looks to signal processing and machine learning. The cognitive process starts with sensing of radio stimuli. The three main cognitive tasks are:

1. Radio scene analysis :

    - Estimation of interference temperature [13].

    - Detection of spectrum holes.

2. Channel identification :

    - Estimation of channel side information (CSI) [13].

    - Prediction of channel capacity which can be used by transmitter.[13]

3. Transmit power control and dynamic spectrum management [13].

The first 2 tasks from the above discussion are carried out in the receiver and the last task is carried out in the transmitter. It is clear that the cognitive receiver and transmitter must always work in sync.

2.3 Shared Spectrum Access

Spectrum sharing is one of the most important advantages a CR system offers. Spectrum sharing is a method which rations the spectrum between a PU and a SU. As discussed, a PU is one who is licensed to used a particular frequency band whereas, SU is one who does not have a license to use a band, but can use it in cooperation with the PU. The CR is able to share spectrum because it intelligently finds and utilizes the spectrum holes in a spectrum. Spectrum sharing can be done in two ways in a CR system, namely: spectrum overlay and spectrum underlay.

- Spectrum overlay: It is a spectrum management principle, in which a SU can access the spectrum for transmission, if and only if, the frequency band is not being used by a PU. It is a very conservative approach. The CR senses the spaces available in a spectrum and then makes the most optimal decision. It is shown in the Figure 2.4 [26].

- Spectrum underlay: In this spectrum management scheme, a PU and an SU can transmit simultaneously given that the SU transmit power is very low and causes negligible interference for the PU transmission. A power threshold has to be established prior to the SU transmission so that the SU does not cross this power level. This threshold is called the IT and will be discussed in detail in the next section. The main metric of a CR which is the QoS (Quality of service) at a PU receiver should not face any disruptive effects due to an SU transmission and this is why IT should be optimally established. However, it should be considered that transmitting a signal with such a low power level

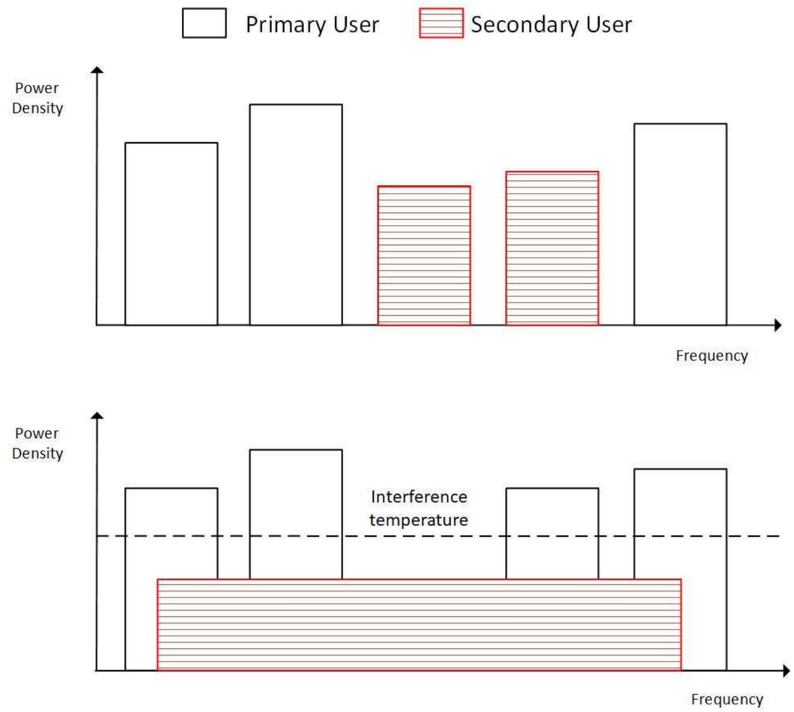

Figure 2.4: Overlay (top) and underlay (bottom) spectrum sharing.

as SU in this transmission scheme, the system can misread the SU signal for noise, causing unwanted issues. Therefore, a well defined space should exist between the defined power threshold level and the noise floor level. Figure 2.4 clearly illustrates the two different spectrum sharing techniques. The upper half of the figure depicts spectrum overlay where SU transmits only when PU is not transmitting. The bottom half shows Spectrum underlay where both PU and SU transmit simultaneously, but the SU power level is considerably lower interference temperature [26].

2.4 Interference Temperature

Interference temperature (IT) is a metric provided by the FCC for interference management in radio environments [27]. Currently, the transmit power of any system is designed to approach a prescribed noise floor level, at a certain distance from the transmitter as shown in the Figure 2.5. However, if the RF noise level increases unpredictably before the prescribed noise floor, it can cause degradation in signal coverage or unintended effects on signal coverage [27, 13]. To avoid such issues, the FCC decided to limit the use of such fixed noise floor levels and encouraged the use of adaptive levels which are based on real time interactions with the transmitter. This adaptive metric is called the IT which deals with all interference sources in the transmitter and radio environment [27]. "IT quantifies and manages interference from various sources" [13]. The main benefits of IT are listed below [13]:

1. It provides an accurate level of interference in the frequency band of interest. Any other transmission which would increase noise levels because of interference could be considered harmful and blocked from transmitting.

2. By quantifying the interference, one could transmit information from other users if the interference limit will not be exceeded.

3. IT can be used as a metric to know whether a SU can transmit on the same band as a PU. If, the transmission of an SU increases the interference more than the IT, the CR system will not allow SU transmission. In this way, IT forms a major metric in a shared spectrum access system, to maintain QoS and efficient communications.

It can also be illustrated from the Figure 2.5. As we can see, the signal noise power approaches to a prescribed noise floor level at the right hand corner [28, 27]. This prescribed noise floor level, can sometime be increased unintentionally, by unwanted sources causing interference. When this happens, the noise floor rises, prematurely cutting the transmit power, hence reducing the signal coverage. To tackle this problem, the FCC introduced adaptive noise floor range or IT [27]. IT is dynamically established in a CR which keeps track of all interference sources and makes sure that no interference would cause an unwanted spike in the noise floor level and degrade communications [28].

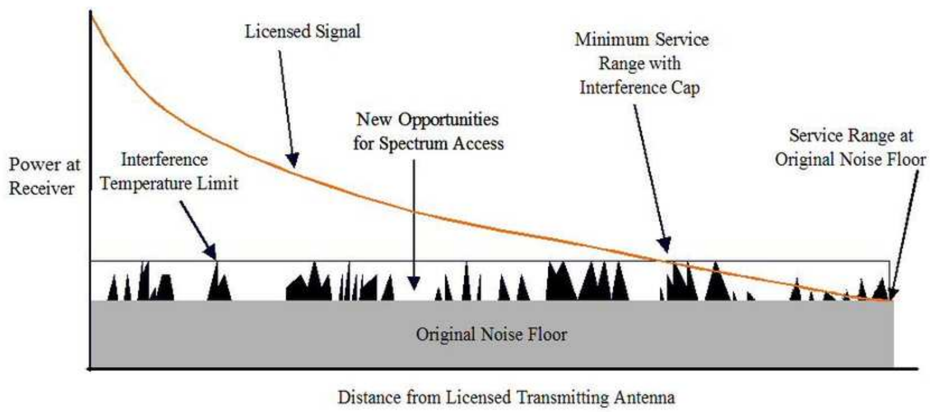

Figure 2.5: Interference temperature [28].

2.5 Power Adaptation

Power adaptation is an important concept utilized in CR systems [29, 30]. It is a method by which the power with which the SU transmits is adapted dynamically to prevent interference at the PU receiver. This is done to maintain the QoS of the licensed users or the PU, which is the main aim of a CR system. If the SU transmit power is higher than the IT, the CR has to limit the power of the SU transmitter or risk causing interference to the PUs. The two main methods of power adaptation are [29]:

- Peak Power Adaptation
- Average Power Adaptation

Peak power adaptation is a scheme in which the peak power transmitted by a SU is always kept lower than the IT. Average power adaptation is a scheme in which the average power transmitted by the SU is limited under IT constraint [29, 30, 22]. Peak power adaptation is more commonly used among the two. Mathematically, they can be expressed as :

$$\mathbb{E}[P(\gamma)] \leqslant \bar{P} \tag{2.1}$$

$$P(\gamma) \leqslant P_{max} \tag{2.2}$$

Where, γ stands for the received SNR. In (2.1), $\mathbb{E}[P(\gamma)]$ stands for the expectation of instantaneous power over time and $\bar{P}$ is the maximum allowed average power. Similarly, in 2.2, $P(\gamma)$ stands for instantaneous peak power and P_{max} stands for maximum permitted peak power. In this way, power adaptation methods help keep SU transmit power under check and hence, control interference of a CR system.

In this chapter, important terms used in CR was described first. Then, detailed description of IT was given. Lastly, power adaptation was studied. In the next chapter, BER will be discussed along with modulation schemes.

CHAPTER 3

BIT ERROR RATE AND MODULATION SCHEMES

The basis of the work done in this thesis is centered around a performance metric called Bit Error Rate (BER). BER is found at the SU under different modulation schemes as will be illustrated in upcoming chapters. Thus, it is very important to understand the concepts of BER and modulation schemes.

3.1 Bit Error Rate (BER)

Whenever data is transmitted over a communication channel, there is a chance of errors being introduced in the data. If a considerable amount of errors are induced during transmission, integrity of the system might be compromised. Thus, it is necessary to monitor the efficiency of a system, and this is where BER becomes an important parameter to monitor system performance [31, 25].

As the name suggests, BER is defined as the rate at which errors occur while transmission of data from a source to destination. If the rate of errors occurring is high, the system is not very efficient. On the other hand, a system is efficient if error rate or BER is low. In other words, let us assume we send a string of words over a wireless communication channel. If, at the receiver, each string received is the same as the string sent, BER is '0' or the communication system is ideal. This, however is difficult to achieve in the real world . BER is defined as:

$$Bit\ Error\ Rate\ (BER) = \frac{Number\ of\ Errors}{Total\ number\ of\ bits\ sent} \tag{3.1}$$

Obviously, a low BER is desired for any system. But, practically, it is very difficult to limit errors, especially in a wireless environment, where degradation of the channel might occur due to various factors like: multipath, shadowing etc [32]. The errors or noise generated in a wireless channel is modelled as a random variable, which generally follows Gaussian probability function while the channel propagation model follows Rayleigh distribution. The characterization of noise and channel environment in mathematical terms, helps us evaluate system performance in a better manner. Also, noise can be added because of inherent behaviour of atoms present in the system elements and such noise is called 'Shot noise', 'Flicker noise' and thermal noise. This kind of noise cannot be restricted and has to be accounted for [33, 34, 35].

BER is a more general parameter for evaluating system performance. But, for communication systems or radio systems, some other parameters like signal to noise ratios and $\frac{E_b}{N_0}$ are more commonly used. Signal to noise ratio (SNR) can be defined as the ratio of strength of signal or useful information to that of interference or noise, and is measured in decibels or dB. For such systems, BER is sometimes called as Probability of Error (PoE).

Energy per bit to noise spectral density ($\frac{E_b}{N_0}$) is another important parameter in digital transmission. It is a normalized measure of SNR which is also defined as the "SNR per bit". It is the most commonly used performance measure to monitor system performance. E_b is defined as the energy present in one bit and N_0 is defined as the noise spectral density or 'power per unit bandwidth'. The energy per bit is power divided with bit rate and its units are joules. Noise spectral density is power per hertz. Thus the units cancel out, leaving BER as a dimensionless quantity. BER plots are one of the most useful tools in wireless communication to give a quick idea about the performance of a system. A BER plot is a plot between BER and SNR per bit or $\frac{E_b}{N_0}$ in dB. A generic BER plot is shown in the Figure 3.1 [36].

Also, it is important to remember that different modulation schemes have different noise performances. Higher order modulation schemes (will be discussed in upcoming sections) have higher data rates, but are more susceptible to noise. Similarly, lower order modulation schemes are more robust in the presence of noise, but have lower data rates. Figure 3.1 shows a typical BER plot for the Quadrature Amplitude Modulation (QAM) scheme, for different orders of modulation M. QAM is a modulation scheme in which the amplitude and the phase of a carrier signal are simultaneously varied with respect to a message signal. It is discussed in detail in Section 3.3.

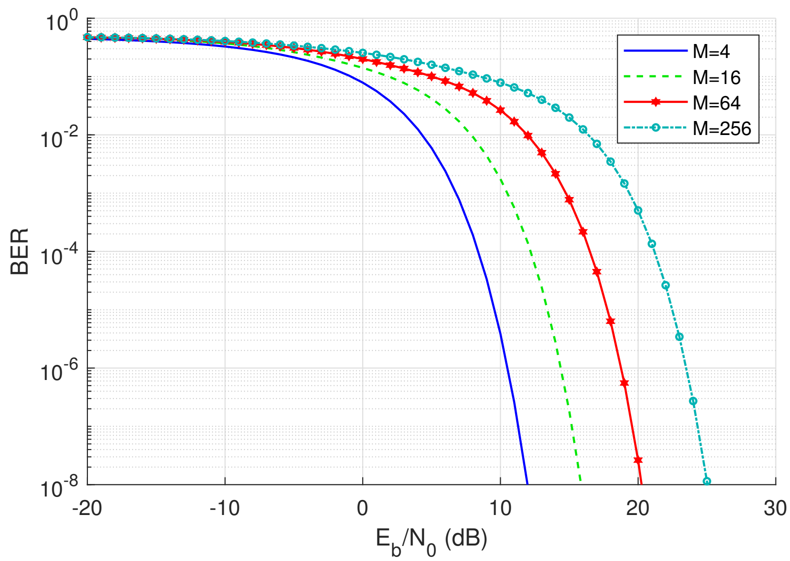

Figure 3.1: Example of a BER plot for M-QAM in AWGN channel.

3.1.1 Factors Effecting BER

As we discussed, BER is directly proportional to $\frac{E_b}{N_0}$ and changing parameters can result in lower or higher BER. Thus, by manipulating these variables, we could optimize a system and is generally undertaken in the design stages of a transmission system [36].

1. Interference: The interference levels have a direct impact on the BER of a system. In most cases, interference is external and cannot be controlled. But, we could window out interference by limiting the bandwidth. This would lead to reduced data rates but, is effective for reducing BER.

2. Increasing transmit power: As mentioned, E_b is energy per bit and BER is directly proportional to this parameter. Naturally, increasing the power per bit would increase the probability of the bits reaching in the right order at the receiver. But, the trade-off would be in terms of battery life, increased power consumption etc.

3. Lower order modulation: Lower order modulation schemes are less susceptible to noise, hence giving desired BER but, at the cost of reduced data rates. A proper trade-off must be found between the desired parameters.

Thus, it is clear that some kind of trade-off is required with the above mentioned factors, to achieve the desired BER. Normally, even though the most satisfactory BER might not be achieved in a system, some other tools called Error Correction Codes (ERC) and Error Detection Codes (EDC) help us in reducing overall errors in the system, hence improving overall BER [37]. In short, BER is a very strong tool which is extensively used in wireless communications. A good knowledge of BER helps us to tailor power and bandwidth according to our requirements, and ultimately, improves efficiency of a system, which is the main goal of a wireless communication engineer [38].

3.2 Modulation Schemes

In any communication block diagram modulation is an integral part of communication as can be seen in Figure 3.2.

Generally, signals generated in a system, by the source are low frequency signals. Since they are low frequency signals, they cannot travel over long distances to reach the receiver and this will lead to attenuation of a signal. Attenuated signals are weak signals, and are more susceptible to noise and interference. This would lead to a dip in the overall performance of a system [37]. To avoid this situation, we use a

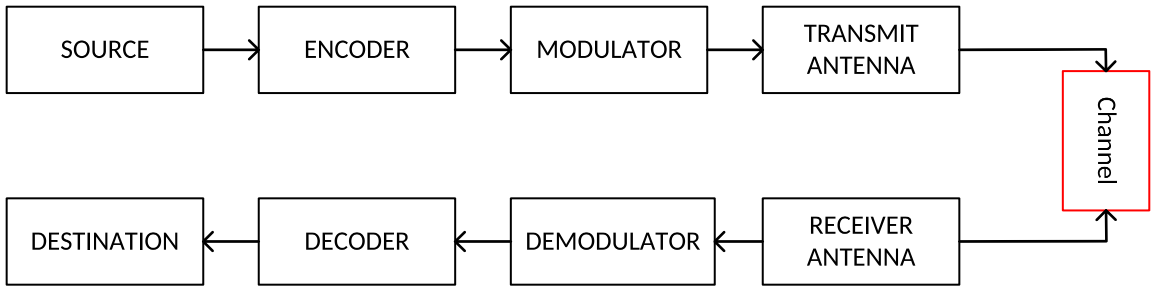

Figure 3.2: Communication block diagram [39].

technique called Modulation. Modulation is defined as a process of varying one or more signal parameters of a periodic waveform (Amplitude, Phase or Frequency), called the carrier signal, with respect to a modulating signal or the signal which contains the information to be transmitted. This means that we multiply a low frequency signal which is to be transmitted over the channel with a signal with varying parameters like amplitude, phase or frequency, to boost the strength of the message or information signal. This will make the information signal stronger and resistant to noise and interference, during transmission over a channel. Also, stronger signals can travel over longer distances. Modulation helps preserve integrity of a communication system [37, 38].

3.2.1 Need for Modulation

- Modulation can transfer the frequency of a signal from one range to another. This would mean that we could transmit a larger number of signals over the available bandwidth, as each signal could have its unique frequency, mitigating interference and improving spectral efficiency [37, 38].

- For efficient transmission, the size of the antenna should at least be $\lambda/4$ times that of the signal being transmitted, λ being the wavelength of the signal. We also know that λ and frequency are inversely proportional to each other. This means, lower the wavelength, higher the frequency and vice-versa. Thus, when a system generates a low frequency signal to be transmitted, it would require a very high antenna to transmit efficiently, which is economically not viable. Hence, modulation is used to convert the low frequency signal into higher frequencies, thus reducing antenna size [37, 38].

- Modulation reduces interference and improves quality of communication.

3.2.2 Types of Modulation

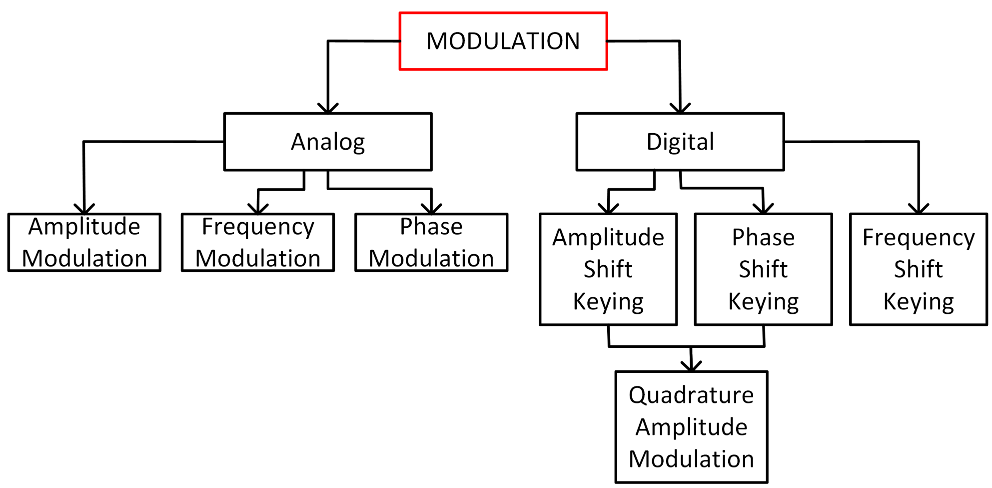

Figure 3.3: Different modulation schemes.

3.2.3 Analog Modulation

All the signals generated in nature are analog signals. Analog modulation is a technique where a carrier wave's parameters are varied in accordance with the input signal to be transmitted. Different analog modulation schemes are:

- Amplitude Modulation (AM): As can be seen from the Figure 3.4, AM is a method where the amplitude or envelope of the carrier signal is changed in accordance to the input signal, while the frequency and phase of the signal remain the same. This technique is susceptible to noise and interference.

- Frequency Modulation (FM): In this technique, frequency of the carrier signal is changed with respect to the carrier signal. Most common used technique in radio and communication fields because of its low susceptibility to noise.

- Phase Modulation (PM): Similarly, in this case, only the phase of the carrier signal is changed in accordance to the input signal.

Figure 3.4 shows different analog modulation techniques:

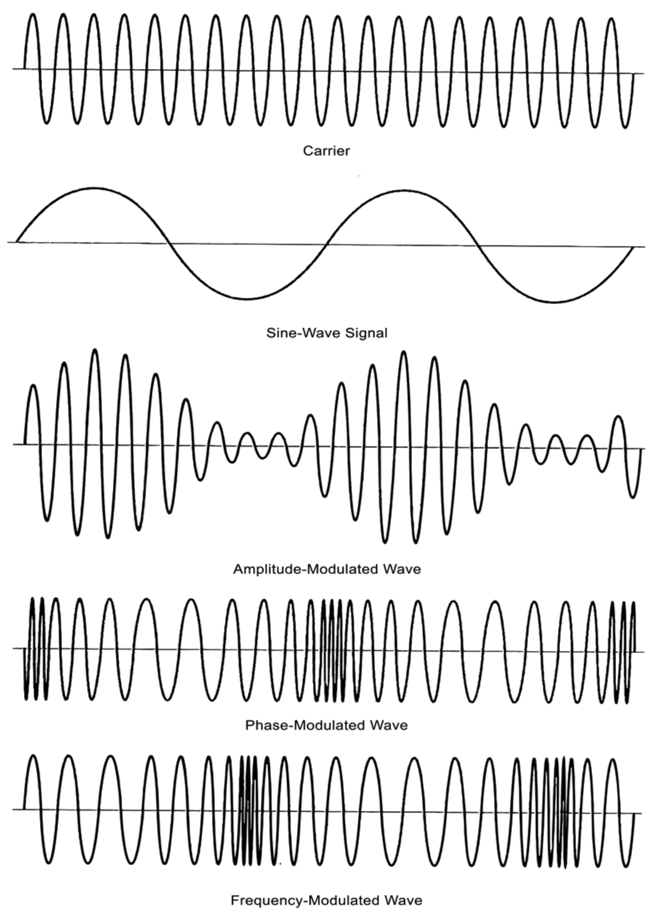

Figure 3.4: Analog modulation [40].

3.2.4 Digital Modulation

Generally, in the past few decades, due to the improvements in digital technologies, digital communication is preferred over analog communication [38] because of the following reasons:

- Digital signals are more resistant to noise.
- Digital signals can be manipulated easily, and the characteristics of the signal can be enhanced or modified in desired forms.
- Hardware is cheaper for digital communications. Also, the digital data can be stored and retrieved easily whenever needed, but it is not the same with analog data.
- Digital signals are more secure because they can be encrypted and are harder to decoded.
- Error correction and error detection codes can be easily implemented for digital signals, which is very useful.

Above mentioned are some advantages of digital communication over analog communication. Hence, the signal generated from a source, which is typically an analog signal, is converted to a discrete signal and then to a digital signal before transmission over the channel. This conversion of analog to digital signal means that analog modulation techniques are no longer useful here, because the signal is now in a different form. Thus, digital modulation techniques were devised to further improve quality of communications. The different types of basic digital modulation schemes are [41]:

- Amplitude Shift Keying (ASK): In this technique, each amplitude, of the output signal depends on whether the input signal bit is a 1 or 0. For example, if the input signal bit is 1, the amplitude of the output signal will be the same as that of the carrier signal, else, the output will be 0.
- Frequency Shift Keying (FSK): In this technique, the frequency of the output signal depends on whether the input signal bit is 1 or 0. If the input bit is 1, the frequency of the output signal will be higher, else, the frequency is lower.

- Phase Shift keying (PSK): In this technique, the phase of the output signal depends on the input data applied.

Figure 3.5 shows the different digital modulation techniques:

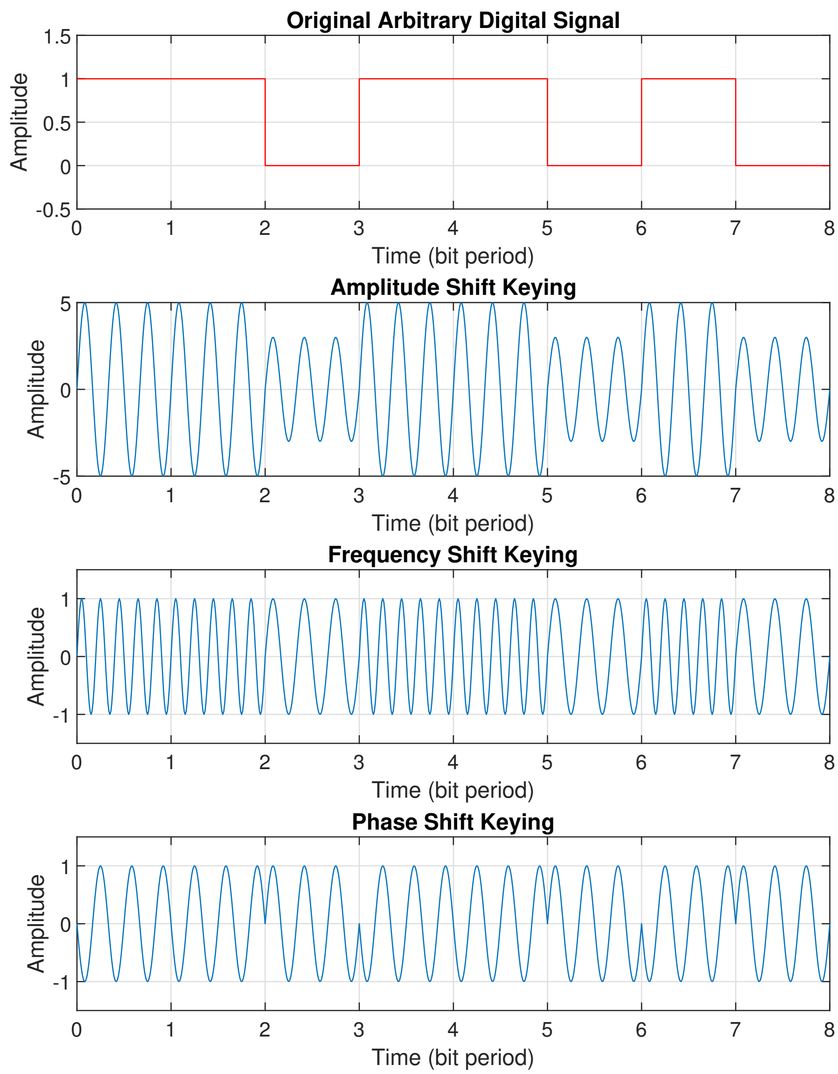

Figure 3.5: Digital modulation [38].

3.2.5 M-ary Transmission

M-ary modulation is a technique in which, instead of transmitting either only a 1 or a 0, bits are grouped together and transmitted simultaneously. Basically, this means we could vary more than just one parameter of a signal [37]. We could vary amplitude, phase and frequency of a signal simultaneously, to produce different combinations of a signal. For example, in ASK, instead of using 1 to represent a certain voltage level and 0 to represent another voltage level, we could group a adjacent 1 and 0 to represent a single voltage level. Consider a data string: 001110101111010. Digitally, we could represent this data stream as one voltage level for 1 and one voltage level for 0. This technique is called Binary modulation. Instead, we could represent a voltage level for 00, 01, 10 and 11. this means we represent one symbol with two bits instead of 1 bit, reducing the total number of bits used to interpret the same data. The number of possible signals is represented by a simple formula : $M = \log(2^n)$, where M is the number of possible signals or different symbols, and n is an integer which defines the number of signals possible. M-ary modulation technique has the following advantages:

- Increased bandwidth efficiency.
- Higher bit rate, for a give symbol rate.
- Reduced bandwidth.

Hence, M-ary signalling techniques are preferred in modern communication [38, 37, 41], because higher the modulation order or M, higher the data rate and lesser the bandwidth used. However, one disadvantage is that higher the modulation order, more susceptible the signal is to noise and external interference as the receiver struggles to interpret and demodulate the signal. Thus, a careful trade-off should be devised between the modulation order, bandwidth requirements and data rate requirements to yield maximum efficiency. The Figure 3.1 explains different bits used to represent a symbol in different modulation orders:

n	$M = 2^n$	Symbol
1	2	$0, 1$
2	4	$00, 11, ...$
3	8	$000, 001, 010, 011, ...$
4	16	$0000, 0001, 0010, 0011, ...$

Table 3.1: M-ary modulation [41].

In the work done in this thesis, we consider the BER of a SU, considering the modulations described in the Table 3.2:

Modulation Scheme	Order of Modulation (M)
Rectangular Quadrature Amplitude Modulation (R-QAM)	$4, 16, 64, 256$
Non-Rectangular Quadrature Amplitude Modulation (NR-QAM)	$8, 32, 128, 512$
Phase Amplitude Modulation (PAM)	$2, 4, 8, 16, 32$
Phase Shift keying (PSK)	$2, 4, 8, 16, 32$

Table 3.2: Modulation scenarios in this thesis.

As established, M-ary transmission opens up a wide range of new possibilities for modulation. In the next section, we will discuss in detail the modulation schemes used in our work, namely, M-QAM, M-PSK and M-PAM.

3.3 Quadrature Amplitude Modulation (QAM)

QAM is one of the most widely used modulation technique in communications because of the advantages it offers, in comparison to other basic modulation techniques. QAM can be achieved when both the amplitude and phase of the carrier signal are varied in accordance to the input signal. The result is that the output signal is represented with considerably lesser number of symbols, hence conserving bandwidth and improving data rate [42, 43].

3.3.1 4-QAM or QPSK

The way in which a QAM signal can be generated is simple. Let us look at the working of a 4-QAM system. The system generates two carrier signals and both carrier signals are out of phase with each other by 90°. The input signal is then, modulated with each of these carrier waves, separately. The two different outputs are then summed together to produce the QAM output. Since, in 4-QAM, we only change the phase of the carrier signal, it can also be called as QPSK or Quadrature Phase Shift Keying. The two out of phase carrier signals are better known as the In-phase component (I) and quadrature component (Q) [41].

$$\begin{aligned} I &= A\cos(\psi) \\ Q &= A\sin(\psi) \end{aligned} \tag{3.2}$$

A constellation diagram shown in Figure 3.6, better explains the concept of QAM. Starting at 45°, the signal phase is incremented 90° to approach 135°, 225° and 315°. Each phase produces a different signal output and this represents different symbols. 2 bits are used to represent a symbol, the symbol rate is reduced by half, thus conserving bandwidth. Table 3.3 summarizes the phases and amplitudes of QPSK [41].

A constellation diagram is a representation of a signal modulated by a modulation scheme. It is a 2-dimensional approach to represent the final modulated signal. The output signal has 4 different phases, which define 4 different symbols, all with the same amplitude level of 1 [32].

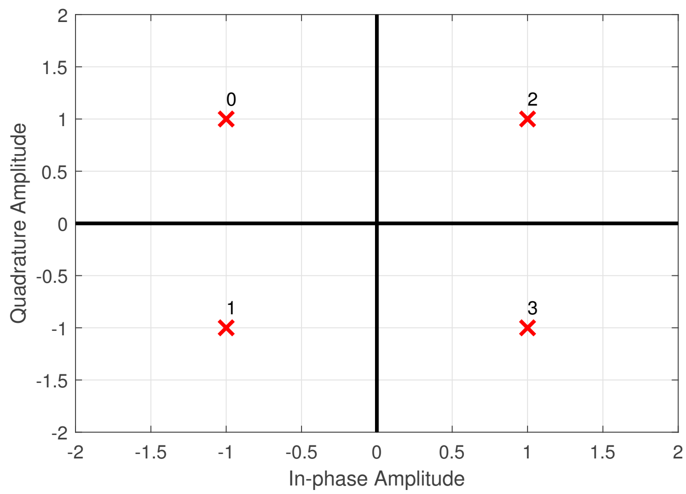

Figure 3.6: Constellation diagram of QPSK or 4-QAM [32].

Symbol Transmitted	Carrier Phase	Carrier Amplitude
00	225°	1.0
01	135°	1.0
10	315°	1.0
11	45°	1.0

Table 3.3: QPSK phase and amplitudes.

3.3.2 16-QAM

As the name suggests, the output of this modulation scheme will have 16 different amplitudes. A constellation diagram for 16-QAM is shown in Figure 3.7. This can be achieved by varying the phase along with the amplitude to get 16 distinct amplitudes. 4 bits represent 1 symbol, in contrary to 4-QAM, where 2 bits represented 1 symbol [41]. It is similar to QPSK, in the sense that 4 different phases are present in the output signal, but, in addition to different phases, 4 intermediate amplitudes are also present, giving us 16 different states. Starting from 15°, the phase is varied in increments of 30°. The constellation diagram and Table 3.4 can better explain 16-QAM [41].

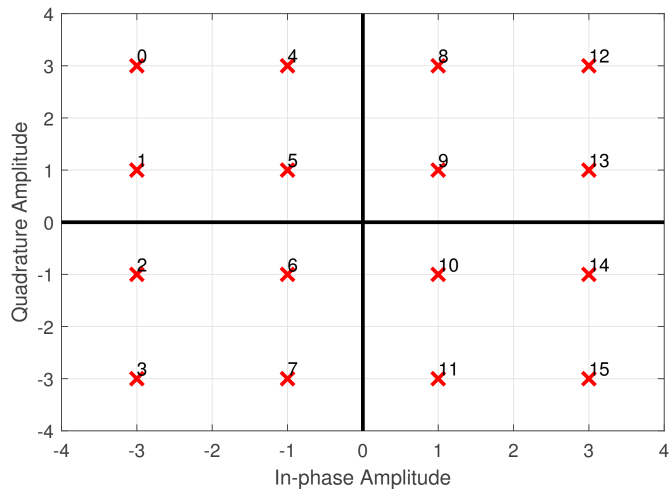

Figure 3.7: 16-QAM constellation diagram [32].

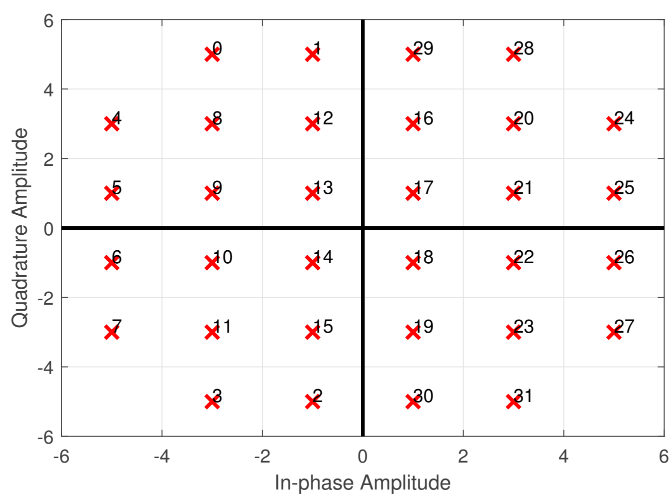

Figure 3.8: Constellation diagram of 32-QAM [32].

Similarly, if we further increase the number of phase shifts and amplitude shifts, we can end up with 64-QAM and 256-QAM.

It can be inferred from the constellation diagrams in Figures 3.6 and 3.7, the constellations are square shaped or rectangular shaped. There are irregular shaped

Symbol Transmitted	Carrier Phase	Carrier Amplitude
0000	225°	0.33
0001	255°	0.75
0010	195°	0.75
0011	225°	1
0100	135°	0.33
0101	105°	0.75
0110	165°	0.75
0111	135°	1
1000	315°	0.33
1001	285°	0.75
1010	345°	0.75
1011	315°	1
1100	45°	0.33
1101	75°	0.75
1110	15°	0.75
1111	45°	1

Table 3.4: 16-QAM phase and amplitudes [41].

constellations too and the modulations are called non-rectangular QAM as in Figure 3.8. This hexagon shaped constellation has better power efficiency in comparison to rectangular ones, but, at the expense of increased complexity in the constellation map [32]. This irregular constellation, can save up to $1.3dB$ of power. Hence,this opened up the field of non-rectangular modulation schemes. Though power efficient, these schemes are complex and hence less preferred.

The following table summarizes the bit rates of different modulation schemes:

Modulation Scheme	Bits per Symbol	Symbol Rate
BPSK	1	1
QPSK	2	1/2
8-PSK	3	1/3
16-QAM	4	1/4
32-QAM	5	1/5
64-QAM	6	1/6

Table 3.5: Bits per symbol comparison [36].

3.4 Phase Shift Keying (PSK)

Phase shift keying (PSK) is another very common modulation technique, which finds use in a wide range of communication applications [44]. It is extensively used in wireless LAN's and has many advantages over Frequency Shift Keying and other techniques [45]. There are a lot of different variations of PSK like [44]:

1. Binary Phase Shift Keying (BPSK)
2. Quadrature Phase Shift Keying (QPSK)
3. Differential Phase Shift Keying (DPSK)
4. Offset Quadrature Phase Shift Keying (O-QPSK)
5. Minimum Shift Keying (MSK)

3.4.1 Binary Phase Shift Keying (BPSK)

It is one of the most basic forms of modulation, and as the name suggests, BPSK is used to represent 2 bits, namely, 1 and 0. The carrier wave is reversed by a phase of 180° to represent a change in bits which is either 1 and 0. This is shown in the constellation diagram Figure 3.9 [44].

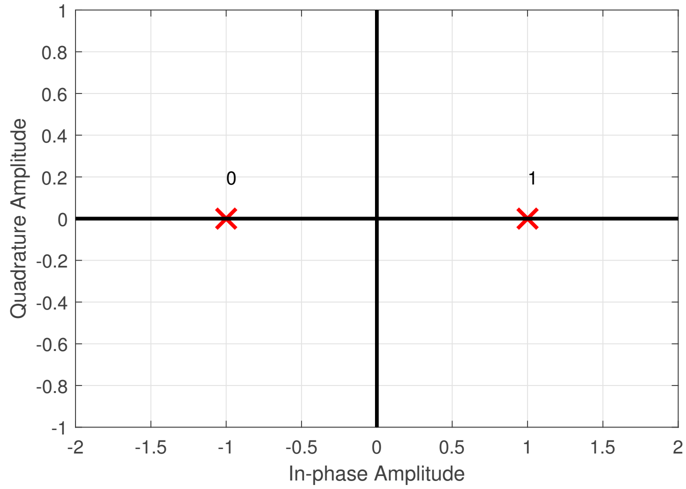

Figure 3.9: BPSK constellation diagram.

When the input bit stream changes from 1 to 0, this is represented in the BPSK waveform, by a 180° change in phase [44]. For this reason, BPSK is sometimes, also referred to as Binary Phase Reversal Keying. The constellation diagram 3.9, shows that the two bits, 1 and 0 are represented by two different phases, namely, 0° and 180°. The working of a BPSK modulator is simple. If the input bit stream is 1, the carrier waveform is a simple sine wave, but when the bit stream is 0, another carrier wave, which is shifted by 180°, is generated by the system. The two carrier waves are added with each other, to give the final BPSK output wave [44].

3.5 Pulse Amplitude Modulation (PAM)

Pulse Amplitude Modulation (PAM) is a simple technique in which, the amplitude of the PAM signal contains information, which directly reflect the information in the message signal [46][38]. In this modulation, a pulse train is generated as a carrier signal. This pulse train is multiplied with the message signal to produce the output waveform which almost resembles the message signal. It is an analog modulation technique and one of the first steps in converting analog to digital signals. The amplitude levels of the output signal are decided by the order of modulation M. If

M is 2, the amplitude levels are 1 and -1. The output signal level can only be either 1 or -1, meaning the input signal would not be perfectly represented as there are only two amplitude levels available. For $M = 4$, the available amplitude levels to represent in the output signal would be ± 1 and ± 3, similarly for $M = 8$, amplitude levels will be ± 1, ± 3, ± 5 and ± 7 and so on for higher orders of modulation. The higher the amount of amplitude levels available, the more accurate and precisely, the receiver can decode and interpret the message signal [46].

Modulation Scheme	Information contained in
Phase Shift Keying (PSK)	Phase
Pulse Amplitude Modulation (PAM)	Amplitude
Quadrature Amplitude Modulation (QAM)	both Amplitude and Phase

Table 3.6: Summary of modulation schemes under consideration.

3.6 Bit Error Rates for Different Modulation Schemes in AWGN Channel

3.6.1 Additive White Gaussian Noise (AWGN) Channel

AWGN is a simple noise model that is used to replicate the noise which is added to the message signal in the channel. The wireless channel is a random environment which adds random noise and interference to the message signal being transmitted. High complexity noise models are also available namely Rayleigh, Rician etc. But AWGN is the most basic and most widely used model [37, 38]. The characteristics of AWGN are as follows:

- Additive: AWGN is additive in nature, meaning the noise is added to the signal rather than being multiplied. The received signal $r(t)$ can be represented as

$r(t) = x(t) + n(t)$, where $x(t)$ is the original signal being transmitted and $n(t)$ is the noise [47].

- White: This means that the noise power stays consistent for all the frequencies [48].

- Gaussian: AWGN is generated due to the thermal noise present in all materials and this kind of noise cannot be avoided. Thermal noise is random in nature and follows the Gaussian distribution. Thermal noise is 0 mean in nature and has a variance of noise power n. 0 mean means that the expected value of $n(t)$ at any time interval is almost 0. The probability of $n(t) = 0$ is very high and the value rapidly decreases as the magnitude is increased [48].

3.6.2 Exact BER Expression for M-PSK

In AWGN, the exact BER expressions for M-PSK can be defined as [49]:

$$P_b(E) = \begin{cases} Q\left(\sqrt{\frac{2E_b}{N_0}}\right), & \text{for} M = 2 \\ \frac{1}{2}(P_1 + 2P_2 + P_3), & \text{for} M = 4 \\ \frac{1}{3}(P_1 + 2P_2 + P_3 + 2P_4 + 3P_5 + 2P_6 + P_7), & \text{for} M = 8 \\ \frac{1}{2}(\sum\limits_{k=1}^{8} P_k + \sum\limits_{k=2}^{5} P_k + P_5 + 2P_6 + P_7), & \text{for} M = 16 \\ \frac{2}{5}(\sum\limits_{k=1}^{16} P_k + \sum\limits_{k=2}^{9} P_k + \sum\limits_{k=5}^{12} P_k - \sum\limits_{k=7}^{14} P_k + \sum\limits_{k=9}^{16} P_k & \\ +2P_{10} + 2P_{11} + P_{12} + 2P_{13} + 2P_{14} - P_{16}), & \text{for} M = 32 \end{cases} \tag{3.3}$$

$$P_k = \frac{1}{2\pi} \int\limits_{0}^{\pi \frac{1-(2k-1)}{M}} \exp \frac{-E_s \sin^2[(2k-1)\pi]}{N_0 \ \sin^2\theta \ M} d\theta - \frac{1}{2\pi} \int\limits_{0}^{\pi \frac{1-(2k-1)}{M}} \exp \frac{-E_s \sin^2[(2k+1)\pi]}{N_0 \ \sin^2\theta \ M} d\theta. \tag{3.4}$$

where $k = 0, 1, 2, ...M-1$, $M = 2, 4, 8, 16$ and 32, P_k is the power of the k^{th} bit, θ is the phase and E_s is the energy per symbol.

3.6.3 Exact BER Expression for M-PAM

The exact BER expressions for M-PAM can be given by [50]:

$$P_b(k) = \frac{1}{M} \sum_{i=0}^{(1-2^{-k})M-1} (-1)^{\left(\frac{i.2^{k-1}}{M}\right)} \left(2^{k-1} - \left(\frac{i2^{k-1}}{M} + \frac{1}{2}\right)\right) erfc\left((2.i+1)\sqrt{\frac{3\log M.\gamma}{M^2-1}}\right). \tag{3.5}$$

where $k = 0, 1, 2, ...M-1$, $M = 2, 4, 8, 16$ and 32, $\gamma = \frac{E_b}{N_0}$ and $erfc$ is the complementary error function.

3.6.4 Exact BER Expression for M-QAM

Exact BER Expression for M-QAM is given as [46]:

$$P_b(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} (-1)^{\left(\frac{i.2^{k-1}}{\sqrt{M}}\right)} \left(2^{k-1} - \left(\frac{i2^{k-1}}{\sqrt{M}} + \frac{1}{2}\right)\right) erfc\left((2.i+1)\sqrt{\frac{3\log M.\gamma}{M-1}}\right) \tag{3.6}$$

where $k = 0, 1, 2, ...M-1$, $M = 2, 4, 8, 16$ and 32, $\gamma = \frac{E_b}{N_0}$ and $erfc$ is the complementary error function. The BER for M-QAM is similar to the BER for M-PAM. In other words, M-QAM can be considered as a combination of two independent PAM signals.

3.7 Approximate Expressions for BER

As one can see from the above established equations that the complexity is high. Therefore, we can use BER approximations which are easier to calculate [32].

Some simulations confirmed that the exact expressions and approximations closely matched each other. However, in our work we will use the exact expressions in the next chapter.

Modulation Scheme	BER approximation
BPSK	$P_b \approx Q(\sqrt{2\gamma})$
QPSK or 4-QAM	$P_b \approx Q(\sqrt{2\gamma})$
M-PAM	$P_b \approx \frac{2(M-1)}{M \log M} Q(\sqrt{\frac{6\gamma \log M}{M^2-1}})$
M-PSK	$P_b \approx \frac{2}{\log M} Q(\sqrt{2\gamma \log m} \sin \frac{\pi}{M})$
M-QAM (rectangular)	$P_b \approx \frac{4}{\log M} Q(\sqrt{\frac{3\gamma \log M}{M-1}})$
M-QAM (non rectangular)	$P_b \approx \frac{4}{\log M} Q(\sqrt{\frac{3\gamma \log M}{M-1}})$

Table 3.7: Approximate BER expressions.

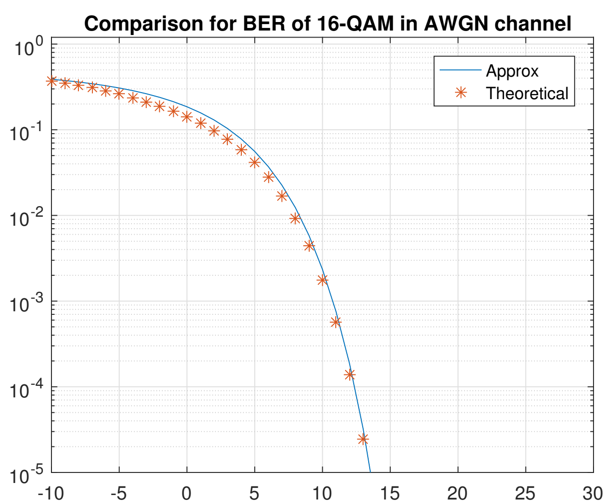

Figure 3.10: Comparison of exact and approx BER for 16-QAM [32].

3.8 Fading Channels

A wireless communication channel is not only susceptible to interference and noise but also phenomenon like fading and path loss. Fading occurs when a transmitted signal reaches the receiver through multiple reflective paths. This causes multipath fading, in which the received signal fluctuates in amplitude, phase and time of arrival [47] [32]. Multiple copies of the same transmitted signal are received at the receiver with different amplitudes and phases causing distortion. We need to quantify the effects of such channels to mitigate its effects and understand how the channel operates [32]. Some such statistical models which help us quantify fading are:

1. Rayleigh fading model
2. Rician fading model
3. Nakagami fading model
4. Weibull fading model
5. Log-normal shadowing model

Rayleigh fading is the most widely accepted fading model and is extensively used for ionospheric and tropospheric propagation's and also for densely packed urban areas. Rayleigh Fading is mostly used when there is no dominant line-of-sight (LoS) propagation occurring between transmitter and receiver [32]. For LoS cases, Rician fading is viewed as the most reasonable model. In fading channels, the fading co-efficient is multiplied with the transmitted signal instead of just being added to the signal. This is unlike the AWGN channel where the noise is added to the signal.

$$y(t) = x(t) * h + n(t) \tag{3.7}$$

where $y(t)$ is the received signal, $x(t)$ is the transmitted signal, h is the Rayleigh fading co-efficient and $n(t)$ is the noise. We only consider Rayleigh Fading because it is the most widely used model [51].

3.8.1 Rayleigh Fading Model

Rayleigh fading model can be used when any transmitted signal passing through a channel is varied randomly, in amplitude and phase according to the Rayleigh distribution [51]. When there are a large number of scattering paths, the Central Limit

Theorem (CLT) states that the channel will be modelled as a Gaussian Random Variable. Whenever the channel is modelled as a zero-mean, Gaussian random process, the channel is said to be Rayleigh fading channel [51]. If it is not zero-mean and a certain component is dominating, it is called Rician fading channel. Consider two multipath components, $x(t)$ and $y(t)$ both zero-mean Gaussian random variables. Simply summing them and taking the envelope of their distribution will give us a Rayleigh distribution [47, 51]. The probability density function(PDF) of Rayleigh distribution is given by:

$$f_x(x) = \frac{x}{\sigma^2} e^{\left(\frac{-x^2}{2\sigma^2}\right)} \tag{3.8}$$

where σ is the standard deviation. Breif analysis confirms that the theoretical and simulated pdf's perfectly match each other as shown in Figure 3.11.

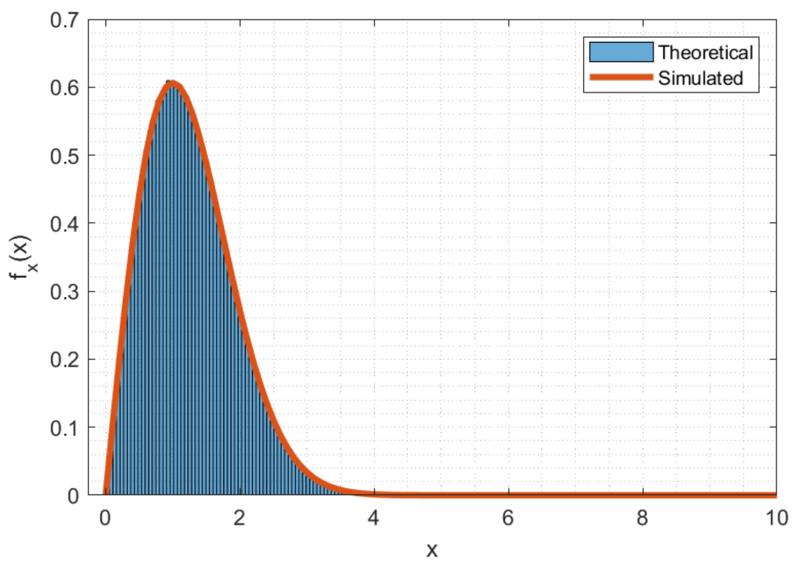

Figure 3.11: Rayleigh distribution.

3.9 Finding BER in Fading Channels

There are a few methods to find BER in fading channels [32]. We take a brief look at two important methods.

3.9.1 Integral Method

The probability of error for a Fading channel is computed by integrating the error probability in AWGN channel over the fading distribution [32].

$$P_b = \int_0^\infty P_b(\gamma) f_\gamma(\gamma) d\gamma \tag{3.9}$$

Here, $P_s(\gamma)$ is the probability of bit error in AWGN channel with SNR γ and $p_{\gamma s}(\gamma)$ is the fading distribution of the channel. Error probability in AWGN channels was discussed in previous sections [32, 47].

3.9.2 Moment Generating Function Approach

The Moment Generating Function approach (MGF) is a useful tool for performance analysis in wireless communications as it greatly simplifies calculations in fading channels [32, 47]. For a random variable γ with distribution $f_\gamma(\gamma)$, the MGF is defined as [32]:

$$M_\gamma(s) = \int_0^\infty f_\gamma(\gamma) e^{s\gamma} d\gamma \tag{3.10}$$

This is just the Laplacian transform of distribution $p_\gamma(\gamma)$ with argument side reversed:

$$\mathcal{L}[f_\gamma(\gamma)] = M_\gamma(-s). \tag{3.11}$$

The MGF of Rayleigh distribution is:

$$M_\gamma(s) = (1 - s\gamma)^{-1} \tag{3.12}$$

Substituting

$$s = -\frac{g}{\sin^2\theta} \tag{3.13}$$

where

$$g = \sin^2\frac{\pi}{M} \tag{3.14}$$

yields,

$$M_\gamma\left(-\frac{g}{\sin^2\theta}\right) = \left(1+\frac{g\gamma_s}{\sin^2\theta}\right)^{-1}. \tag{3.15}$$

This chapter started with the definitions of BER and then looked at some factors which affect BER. Then, modulation was defined, types of modulations, M-ary modulation techniques was looked at in detail. Exact and approximate equations for BER of various modulation schemes was shown. Finally, methods to find BER in fading channels was discussed.

CHAPTER 4

BER ANALYSIS OF SECONDARY USER

4.1 Introduction

In this chapter, the BER performance of a SU in a Cognitive Radio Network (CRN) with interference spreading (IS) is investigated. IS is a technique which is used to uniformly spread the interference of a SU, among the subcarriers. IS method, unlike spectrum sensing (SS), does not need to monitor subcarrier occupation at all times making it highly power efficient than the latter [22, 20, 9]. In addition, IS is less complex due to the low co-operation required between the base stations. Hence the IS method is the preferred mechanism for interference management in this thesis. In the absence of SS, allocation of subcarriers by a base station is assumed to be random [30]. In the following sections, a detailed analysis of SU performance is presented for various modulation schemes. BER is the chosen metric for performance. The main contribution of this thesis is a full detailed error performance analysis of a SU under different modulation schemes with the employment of IS method. Furthermore, the relationship between BER and IT is also investigated.

4.2 Interference Spreading and The Limitations of Spectrum Sensing

In this section, we take a look at some limitations of the commonly used SS mechanism. We then proceed to take a look at an alternative, called IS method and how it has many more advantages over the former mechanism.

Most of the literature survey on CRN shows the heavy dependency on SS [9].

SS is used to monitor the spectrum for holes and free spaces. However, it has a few limitations. From [22], it can be seen that SS provides unreliable spectrum information due to the following:

- Uncertainties due to channel randomness at the devices.
- Hidden primary user problem.
- Decision fusion in cooperative sensing.

Furthermore, SS has to be active at all times, monitoring the spectrum. This consumes a lot of power and hence will not be viable for power limited systems. SS is also a complex mechanism, as it requires a high level of co-operation between the transmitters and receivers. Hence, in this work, we avoid the use of SS. Due to a lack of SS, it is assumed that a random allocation method is deployed to allocate subcarriers [30]. This random allocation causes subcarrier collision or interference with a certain probability. Figure 4.1 illustrates the collisions occurring due to random access with 3 PUs and 1 SU. To manage and limit these collisions or interference, we use a mechanism called IS.

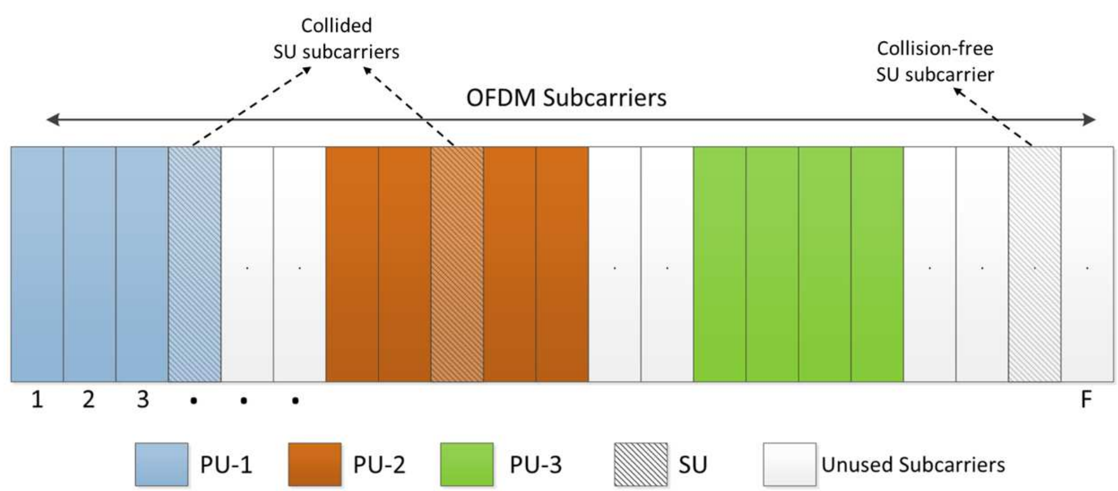

Figure 4.1: Example of collisions due to random access with 3 PUs and 1 SU [52].

The main aim of IS is to spread the interference of the SU, uniformly to all the available subcarriers in the network [52]. This type of IS has a main advantage that no single PU performance will be severely degrade due to interference, instead interference will be distributed uniformly among different subcarriers. This is also the main contribution of this thesis as all the previous work on CRN has been done with a SS mechanism in place. It can be better illustrated by Figure 4.2

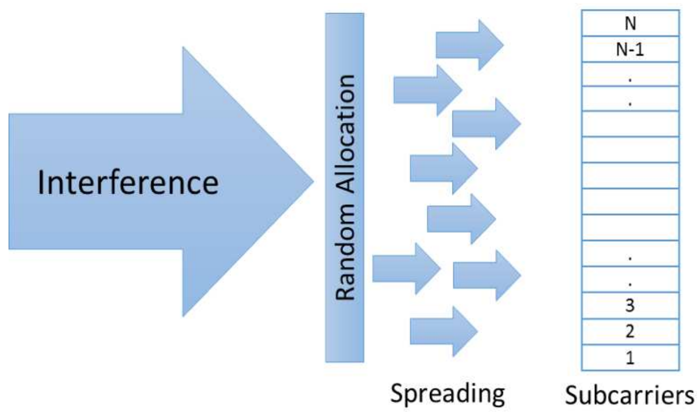

Figure 4.2: Illustration of interference spreading (IS) [52].

To summarize, IS can provide the following functions:

- Uniformly distributes the SUs interference among the PUs subcarriers and free subcarriers.
- Removes the need for SS.
- Power efficient as there is no need for nonstop monitoring of the spectrum.
- Less complex due to the low cooperation between base stations.

It is assumed that the CRN is operating without a SS mechanism. Also, power adaptation is absent and there is no scheduling or rationing of sub-carriers. Let us consider that the CR system has 5 PU's, PU-N where $N = 1, 2, 3, 4, 5$. Since there is no information about PU activities at the SU, SU will randomly allocate sub-carriers which might collide with PU's subcarriers. It is very probable that certain PU's capacity suffer more damage than other PU's. This can be illustrated by Figure 4.3.

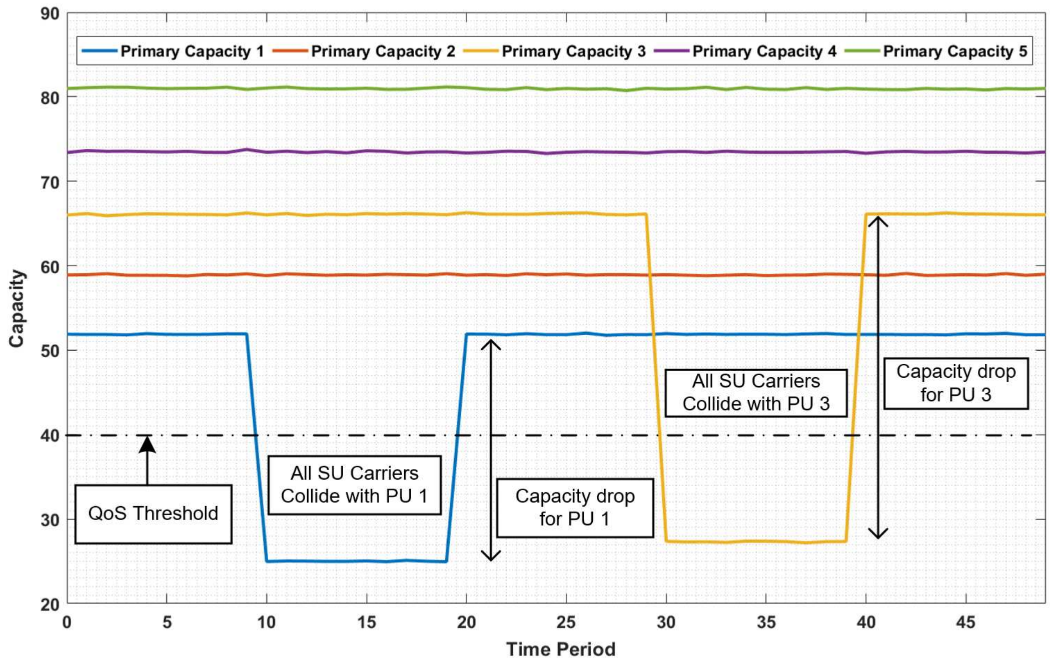

Figure 4.3: Capacity of users without interference spreading [22].

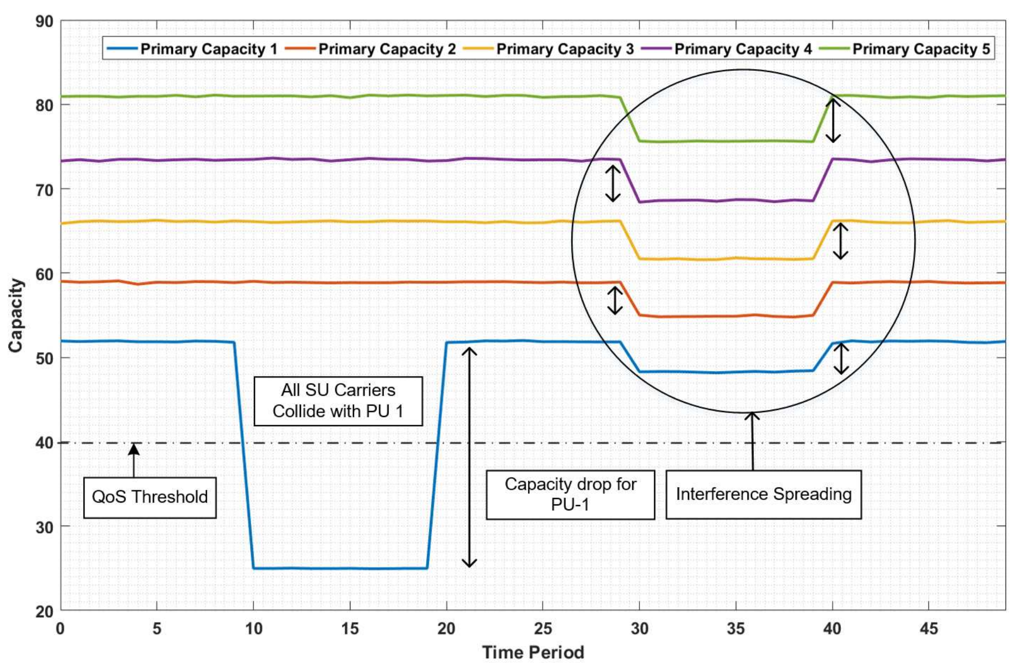

Figure 4.4: Capacity of users with interference spreading [22].

In Figure 4.3, it is seen that PU-1 and PU-3 suffer most of the damage due to SU interference. The capacities for user 1 and 3 take a drastic hit, where the capacity falls below the threshold capacity. Now, using peak power adaptation and uniform scheduling is considered. Under such circumstances, as the results from Figure 4.4 show, there is a huge improvement in individual capacities of users. It can be seen that even though the overall effect of interference is the same in both cases, interference is spread evenly among all users in Figure 4.4. This spreading of interference ensures that no one user suffers more damage than others, which improves the overall efficiency and Quality of Service (QoS) of the CRN. All users are affected evenly, without much harm to the overall system performance, unlike in Figure 4.3 where PU-1 and PU-3 suffer fatal damage. Thus, the use of IS improves performance for the PU, but at the same time, it is assumed that the SU is still under the IT constraint to prevent QoS degradation.

4.3 System Model

The system model is shown in the Figure 4.5 [30]. It consists of a primary base station (PBS) and a secondary base station (SBS). There are P PUs and S SUs.

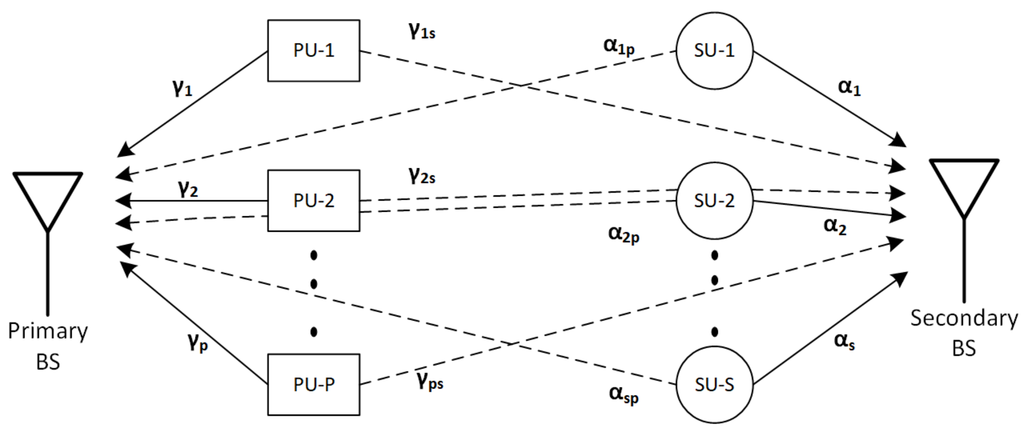

Figure 4.5: System model [30].

The communication between PUs communicate with the PBS and is shown in Figure 4.2 with a solid link. Similarly SU communication with the SBS is shown with a solid link. PU communication with the SBS and SU communication with the PBS are denoted by dotted links. The channel gains are defined as:

- Channels between PU and PBS: $\gamma_1, \gamma_2.........\gamma_p$
- Channels between PU and SBS: $\gamma_{1s}, \gamma_{2s}.........\gamma_{ps}$
- Channels between SU and SBS: $\alpha_{1p}, \alpha_{2p}.........\alpha_{sp}$
- Channels between SU and PBS: $\alpha_1, \alpha_2.........\alpha_s$

It is also assumed that the network is an orthogonal frequency-division multiplexing (OFDM) based cellular network in which the frequency bands are divided into non-overlapping frequency bands which are known as subcarriers. This makes sure that no collisions occur among allocated PU subcarriers. The only collision occurring in the CRN will be when SU is allocated the same subcarriers which are also allocated for the PU [22].

Unit mean independence and Rayleigh fading characteristics are assumed for all channels. The channel power gains are exponentially distributed. It is also assumed that precise information about channel gains γ_p and α_s is available to the SU, but information about PU channel occupation is not available [30]. Channel information can be acquired through a process called Channel Side Information (CSI) through various ways like channel reciprocity condition, by the use of a mediate band or CR network manager between PBS and SBS [53]. The Additive White Gaussian Noise(AWGN) at the PU and SU is assumed to be Gaussian distribution with zero mean and variance η . For further simplicity, the interference temperature Ψ, is kept constant for all subcarriers. It is needed protect the primary network from interference and maintain QoS.

4.3.1 System Model for a Single PU and SU

For initial evaluations of SU BER, a simple network of only one PU and one SU is assumed. This is shown in the Figure 4.6. Collisions occur because of the random allocation of subcarriers. The channel gains for the PU link is given by γ, and the interference link of PU onto SU is given by γ_{ps}. The SU channel gain is given by α and the interference channel gain is given by α_{sp}. The interference which is caused by the SU on the PU is controlled by a parameter called interference temperature or ψ and peak power adaptation is employed to dynamically adapt the power of SU transmitter [30]. This is to preserve the QoS of the PU, which is the main aim of any communication system. Power adaptation process makes sure that the SU power is always under check and hence limiting interference and improving QoS.

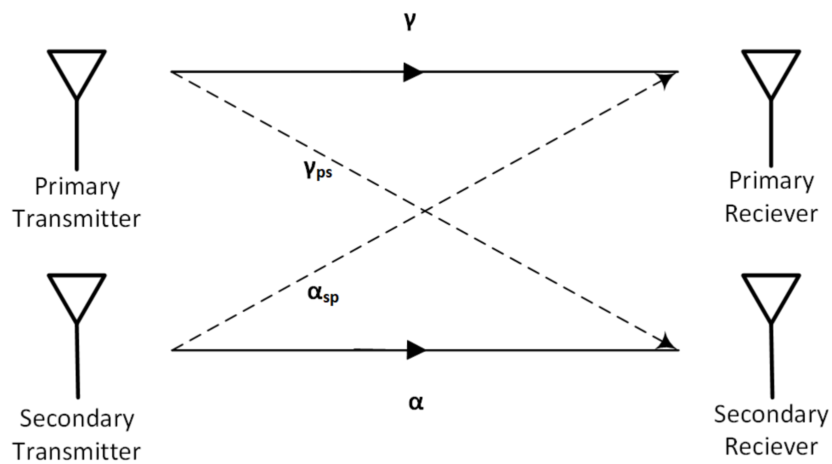

Figure 4.6: System model of a single PU and single SU CR system.

4.4 Math Preliminaries

In this section, first some math preliminaries are established along with the underlying assumptions for the mathematical analysis and then a detailed description of IS method is presented.

4.4.1 System Parameters

It is assumed that there are p number of primary users and s number of secondary users. F_p subcarriers is in use by the pth PU from the pool of F subcarriers. Thus, the number of idle subcarriers are $F - F_p$. Since there is no knowledge of the PU subcarrier usage information, the SU might randomly allocate subcarriers from the in-use PU subcarriers (F_p). The sth SU randomly utilizes F_s subcarriers. Thus, out of F bands, F_p bands are randomly being used by the PU and F_s bands are randomly used by the SU which leads to c_{ps} number of collisions. Interference and collisions are interchangeably used in this work. From [30], the Table 4.1 provides a clear summary of the system parameters:

Symbol used	Description
p	Total number of PUs present
s	Total number of SUs present
F	Total pool of subcarriers available
F_p	Subcarriers allocated to the PU
F_s	Subcarriers allocated to the SU
C_{ps}	Subcarrier collisions occurring
$F - F_p$	Unreserved Subcarriers
C_{non} or $(F_s - C_{ps})$	Free Subcarriers

Table 4.1: System parameters.

Probability of Mass Function (PMF) of Subcarrier Collisions

Probability mass function is a function that gives the probability that a discrete random variable is exactly equal to some value. It is the primary means of defining a discrete random variable. It is analogous to Probability Density Function (PDF) with the only difference that PDF is used in the continuous domain while PMF is used in the discrete domain. Assuming quantities from Table 4.1, the PMF of subcarrier collisions C_{ps} can be found by using a discrete probability distribution called Hypergeometric Distribution. "Hypergeometric Distribution describes the probability of k successes (random draws for which the object drawn has a specified feature) in n draws, without replacement, from a finite population of size N that contains exactly K objects with that feature, wherein each draw is either a success or a failure" [30, 54].

Probability of subcarrier collision is given by:

$$\Pr(C_{ps} = c_{ps}) = p(c_{ps}) = \binom{F}{F_s}^{-1} \binom{F_p}{c_{ps}} \binom{F - F_p}{F_s - c_{ps}}, \tag{4.1}$$

where the notation $\binom{\cdot}{\cdot}$ stands for the binomial coefficient. The average number of subcarrier collisions can be given by:

$$\mathbb{E}[C_{ps}] = \frac{F_s F_p}{F} \tag{4.2}$$

4.4.2 Instantaneous Error rate of SU

As discussed, BER is the metric for performance analysis in this thesis. The instantaneous error rate of the s^{th} SU using F_s subcarriers is given by [30] :

$$P_{e_i} = \frac{1}{F_s} \sum_{i=1}^{F_s} P_e(i). \tag{4.3}$$

where, P_{e_i} is the instantaneous probability of bit error.

4.4.3 Mean Error Rate of SU

As defined, c_{ps} is the number of collisions occurring. We can define the free secondary sub carriers without collisions $(F_s - c_{ps})$ as c_{non}. Similar to (4.2), the average number of non-collisions can be defined as:

$$\mathbb{E}[c_{non}] = \frac{F_s(F - F_p)}{F} \tag{4.4}$$

Mean error rate can be obtained by summing the instantaneous errors for both collision and no collision cases [30]. Mean error rate of a SU using F_s subcarriers is given by:

$$P_e = \frac{1}{F_s} \left\{ \sum_{i=1}^{c_{ps}} P_{b,c} + \sum_{j=1}^{c_{non}} P_{b,nc} \right\} \tag{4.5}$$

where, P_c is the probability of collisions and P_{nc} is the probability of no-collisions. Alternatively, the average probability of error of the SU can be given as [30]

$$
\begin{aligned}
\mathbb{E}\left[P_e\right] &= \mathbb{E}\left[\frac{1}{F_s}\sum_{i=1}^{C_{ps}} P_{b,c} + \frac{1}{F_s}\sum_{i=1}^{C_{non}} P_{b,nc}\right] \\
&= \frac{1}{F_s}\mathbb{E}\left[\mathbb{E}\left[\sum_{i=1}^{C_{ps}} P_{b,c}\middle| C_{ps} = c_{ps}\right]\right] + \mathbb{E}\left[\mathbb{E}\left[\sum_{i=1}^{C_{non}} P_{b,nc}\middle| C_{non} = c_{non}\right]\right] \\
&= \frac{1}{F_s}\mathbb{E}\left[\sum_{i=1}^{c_{ps}} \mathbb{E}\left[P_{b,c}\right]\right] + \mathbb{E}\left[\sum_{i=1}^{c_{non}} \mathbb{E}\left[P_{b,nc}\right]\right] \\
&= \frac{1}{F_s}\mathbb{E}\left[c_{ps}\mathbb{E}\left[P_{b,c}\right]\right] + \mathbb{E}\left[c_{non}\mathbb{E}\left[P_{b,nc}\right]\right],
\end{aligned}
$$

Furthermore, c_{ps} and P_c are independent and so are c_{non} and P_{nc}. Therefore average P_e can be expressed as:

$$
\mathbb{E}[P_e] = \frac{1}{F_s}\big\{\mathbb{E}[c_{ps}]\mathbb{E}[P_{b,c}] + \mathbb{E}[c_{non}]\mathbb{E}[P_{b,nc}]\big\} \tag{4.6}
$$

where c_{ps} and c_{non} are defined in Table 4.1, and $P_{b,c}$ and $P_{b,nc}$ will be found in the upcoming sections.

4.5 BER Analysis of SU over Rayleigh Fading Channel

In this section, the BER of SU over a Rayleigh fading channel is derived. Both collision and no-collision cases are considered to finally arrive at the mean BER of the SU. Peak power interference constraint in used and the transmit power of SU is adapted to maintain the QoS of PU. Hence, the transmit power of the s^{th} SU user for an arbitrary i^{th} subcarrier is given by [29]:

$$
P_s = \begin{cases} P_s, & \psi \geqslant P_s\gamma_{sp} \\ \frac{\psi}{\gamma_{sp}}, & \psi < P_s\gamma_{sp} \end{cases} \tag{4.7}
$$

$$
= min\left\{P_s, \frac{\psi}{\gamma_{sp}}\right\} \tag{4.8}
$$

This means that the power with which the SU transmits is the minimum of its original transmit power P_s or the adapted power $\frac{\psi}{\gamma_{sp}}$. If the transmit power of SU is lower than the pre-defined ψ value, it can transmit with its original power. Else, if the power of SU crosses the threshold value ψ, its power is adapted and changed to $\frac{\psi}{\gamma_{sp}}$ to limit interference and preserve QoS.

Now, let us take a look at the Signal to Interference and Noise Ratio (SINR) of the s^{th} SU's i^{th} subcarrier. SNR is the ratio of received power with the noise. Let $\lambda = P_s\gamma$ and SNR be denoted as S_s. For the collision or interference case, [30]

$$S_{s,c} = \frac{\lambda}{I_p + \eta}, \tag{4.9}$$

where, $p = 1, ...P$. Here, I_p denotes the interference caused by the p^{th} PU onto the i^{th} subcarrier. If interference is present or collisons are occuring, SNR is called as SINR or signal to interference and noise ratio. If there is no collisions or no interference caused by the PU, i.e the subcarrier is only being used by the SU currently and the variable I_p will reduce to 0. SNR will be given by: [30]

$$S_{s,nc} = \frac{\lambda}{\eta} \tag{4.10}$$

The Cumulative Distribution Function (CDF) of λ can be obtained as [55, 30]:

$$F_\lambda(x) = F_{\gamma_{sp}}(\frac{\psi}{P_s})F_{v_1}(x) + F_{v2|\gamma_{sp}>\frac{\psi}{P_s}}(x|\gamma_{sp}>\frac{\psi}{P_2}) \tag{4.11}$$

where, $v_1 = \gamma P_s$ and $v_2 = \frac{\psi\gamma}{\gamma_{sp}}$ and their individual PDF's can be expressed as :

$$f_{v1}(x) = \frac{e^{-\frac{x}{P_s}}}{P_s} \tag{4.12}$$

$$f_{v2}(x) = \frac{\psi}{(x+\psi)^2} \tag{4.13}$$

Thus, the CDF of (4.10) after substituting (4.11) and (4.12) is [30]:

$$F_\lambda(x) = \left(1 - e^{\frac{-\psi}{P_s}}\right)\left(1 - e^{-\frac{x}{P_s}}\right) + e^{-\frac{x}{P_s}} - \frac{-\psi}{P_s + x}e^{-\frac{x+\psi}{P_s}} \tag{4.14}$$

$$= 1 - e^{-\frac{x}{P_s}} + \frac{x}{\psi + x}e^{-\frac{x+\psi}{P_s}} \tag{4.15}$$

The PDF of (4.10) is:

$$f_\lambda(x) = \frac{dF_\lambda(x)}{dx} \tag{4.16}$$

$$= \frac{e^{-\frac{x}{P_s}}}{P_s} \left[1 - e^{-\frac{\psi}{P_s}} \left(\frac{x^2 + \psi x - \psi P_s}{(\psi + x)^2}\right)\right] \tag{4.17}$$

For no collisions case, following (4.10) and using the transformation of random variables: $f_{s,nc}(x) = \eta f_\lambda(\eta x)$ in (4.16), we get the **PDF of the no-collision case as** [30]:

$$f_{s,nc}(x) = \frac{\eta e^{-\frac{\eta x}{P_s}}}{P_s} \left[1 - e^{-\frac{\psi}{P_s}} \frac{((\eta x)^2 + \psi \eta x - \psi P_s)}{(\psi + \eta x)^2}\right] \tag{4.18}$$

Now, from [30], we proceed to derive the PDF of the collision or interference case. By using transformation of Random Variables, the PDF of $S_{s,c}$ with $f_{I_p}(y) = \frac{e^{\frac{y}{P_p}}}{P_p}$ (where I_p is the interference parameter and P_p is the power from the primary transmitter, received at the secondary receiver) can be expressed as :

$$F_{s,c} = P(\lambda < x(I_p + \eta)) \tag{4.19}$$

$$= \int_0^\infty F_\lambda(x(y + \eta)) f_{I_p}(y) dy \tag{4.20}$$

Plugging (4.19) into equation (4.14), we get

$$F_{s,c}(x) = 1 - \frac{\left(1 - e^{-\frac{\psi}{P_s}}\right)\left(e^{-\frac{x\eta}{P_s}}\right)}{1 + \frac{xP_p}{P_s}} - \frac{\psi}{xP_p} \left(e^{\frac{\psi}{xP_p} + \frac{\eta}{P_p}}\right) \Gamma\left(0, \left(\eta + \frac{\psi}{x}\right)\left(\frac{1}{P_p} + \frac{x}{P_s}\right)\right) \tag{4.21}$$

where the incomplete Gamma function is defined as $\Gamma(x, y) = \int_y^\infty t^{x-1} e^{-t} dt$ and derivative of the CDF or equation 4.20 yields the **PDF of collision case as** [30]:

$$f_{s,c}(x) = \frac{x\eta P_p + P_s(\eta + P_p)}{(xP_p + P_s)^2} \left(e^{\frac{\psi}{P_s}-1}\right)\left(e^{-\frac{x\eta+\psi}{P_s}}\right) + \frac{\psi}{x^3 P_p^2} \left(e^{\frac{x\eta+\psi}{xP_p}}\right) \Bigg[(\psi + xP_p)$$
$$\Gamma\left(0, \left(\eta + \frac{\psi}{x}\right)\left(\frac{1}{P_p} + \frac{x}{P_s}\right)\right) + \frac{xP_p(x^2 \eta P_p - \psi P_s)}{(x\eta + \psi)(xP_p + P_s)} - \left(\eta + \frac{\psi}{x}\right)\left(\frac{1}{P_p} + \frac{x}{P_s}\right)\Bigg] \tag{4.22}$$

In summary, the PDF for the collision case and no-collision case have been obtained respectively in the form of equation (4.22) and (4.18). We can now substitute these equations in equation (3.9) to find out the BER of the CR system. We know $P_b = \int_0^\infty P_s(\gamma) P_{\gamma s}(\gamma) d\gamma$ where P_b is the probability of bit error or BER, $P_s(\gamma)$ is the probability of the bit error of the modulation scheme under consideration in AWGN environment and $P_{\gamma s}(\gamma)$ is the PDF of the fading channel over which the BER is found. After substituting (4.22) and (4.18) in (3.9) , 2 new equations are obtained:

BER of no collision or no interference case:

$$P_{b,nc} = \int_{0}^{\infty} P(x)_{AWGN}\ f_{s,nc}(x)dx \tag{4.23}$$

BER of collision or interference case:

$$P_{b,c} = \int_{0}^{\infty} P(x)_{AWGN}\ f_{s,c}(x)dx \tag{4.24}$$

Where, $P(x)_{AWGN}$ was discussed in Section 3.6 of Chapter 3 and $f_{s,nc}(x)$ and $f_{s,c}(x)$ have been established as equations (4.18) and (4.22) respectively.

4.6 Results for BER analysis of SU

The BER plots for the no-collision case are obtained using (4.23) and (4.18). Similarly, BER plots for the collision case are obtained using (4.24) and (4.22). The mean BER is obtained using (4.6). For the above two cases, BER is found for 4 different modulation schemes namely: rectangular M-QAM, non-Rectangular M-QAM, M-PSK and M-PAM.

4.6.1 Plots for No-Collision Case

The BER plots for the no-collision case are obtained using (4.23) and (4.18). The plot for Rectangular M-QAM (M = 4, 16, 64 and 256) is shown in Figure 4.7.

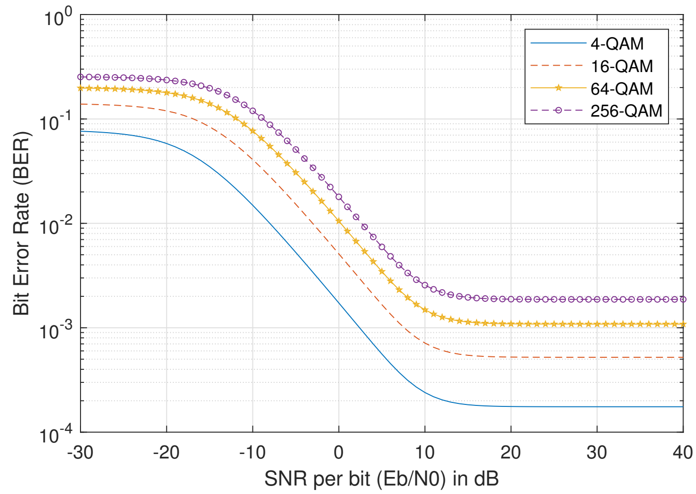

Figure 4.7: BER of M-QAM for no-collision case.

As the order of modulation M increases, the BER also increases which is expected. BER is lowest when $M = 4$ and is highest when $M = 256$. Also, as the SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations.

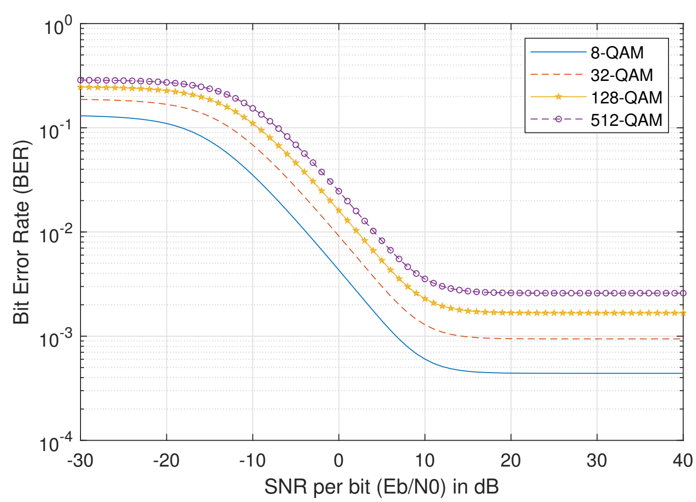

Figure 4.8: BER of non-rectangular M-QAM for no-collision case.

The same can also be seen for non-Rectangular M-QAM (M = 8, 32, 128 and 512) as in Figure 4.8, here, BER is lowest for $M = 8$ and highest for $M = 512$. The trends are in line with expectations.

A plot for M-PSK (M = 2, 4 ,8, 16 and 32) is shown in figure 4.9.

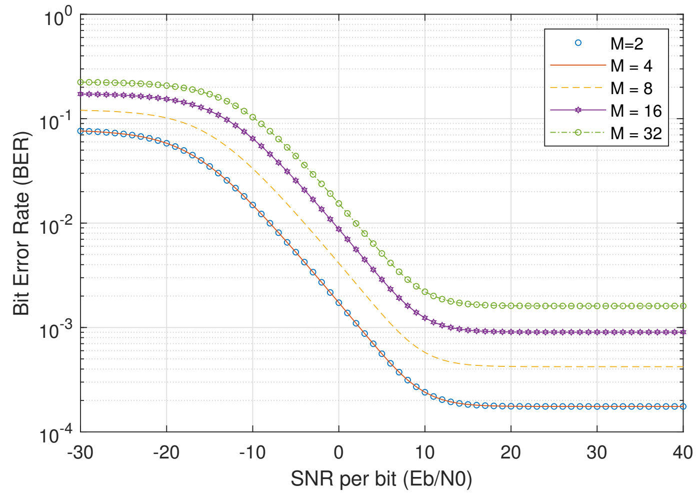

Figure 4.9: BER of M-PSK for no-collision case.

For M-PSK, from Figure 4.9, as the order of modulation M increases, the BER also increases which is expected. BER is lowest when $M = 4$ and is highest when $M = 32$. Also, as the SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations. The BER curve of M-PSK in Figure 4.9 with $M = 2$ and $M = 4$ exactly match each other. This is expected because the BER performance of BPSK (M=2) and QPSK (M=4) are the same [32].

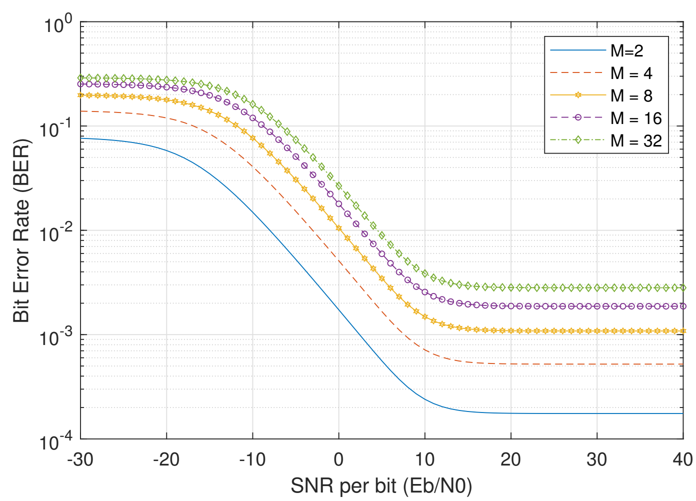

Figure 4.10: BER of M-PAM for no-collision case.

A plot for MPAM (M = 2, 4, 8, 16 and 32) is shown in Figure 4.10. As the order of modulation M increases, the BER also increases which is in line with expectations. Also, as the SNR increases, the BER decreases which matches perfectly with expectations.

A plot QAM, PSK and PAM with M=4 is shown in Figure 4.11. The trend is in line with expectations perfectly as 4-QAM and 4-PSK curves match each other. It is also expected that PAM will have a higher BER than QAM and QPSK. Therefore trend of the curves in the plot make perfect sense [32, 47].

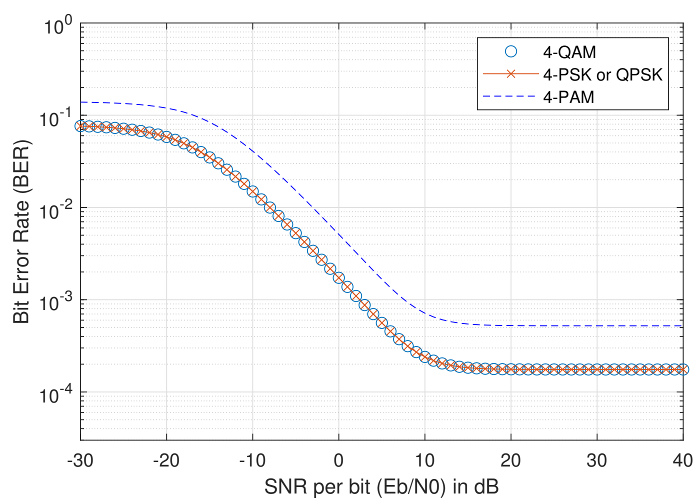

Figure 4.11: Comparison of BER for different Modulation schemes with M=4.

The above plots were obtained for the no-interference case using (4.23). In (4.18), η is 0.01, P_s is varied between $-30dB$ to $40dB$ and ψ is considered as $10dB$. The above plots stick to the desired trend in that, as the SNR improves, the BER decreases. This shows that the system performs better with high SNR as the number of errors are low. The BER remains constant for about -30 to $-10dB$ and the gradually decreases until almost $10dB$. The BER then saturates again and remains constant for the rest of the SNR, meaning that even if the power of SU is increased further, the BER does not improve. This is due to power adaptation because power adaptation makes sure the power of the SU does not increase beyond a certain limit. The IT constraint limits the increase of SU power and hence the BER curve saturates. It can also be seen that for all plots, the BER curves saturate after $10dB$. This makes sense because the IT was considered as $10dB$ for the above cases.

4.6.2 Plots for Collision Case

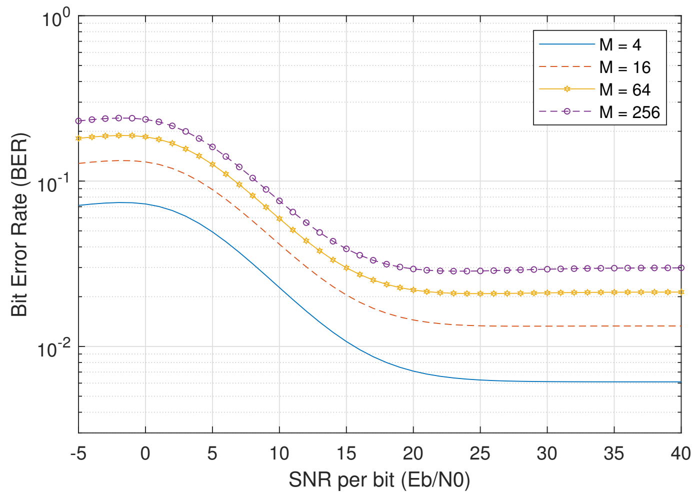

Figure 4.12: BER of Rectangular M-QAM for collision case.

The BER plots for the collision case are obtained using (4.24) and (4.22). The plot for Rectangular M-QAM (M = 4, 16, 64 and 256) is shown in Figure 4.12. As the order of modulation M increases, the BER also increases which is expected. BER is lowest when $M = 4$ and is highest when $M = 256$. Also, as the SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations. The same can also be seen for non-rectangular M-QAM (M = 8, 32, 128 and 512) as in Figure 4.13, here, BER is lowest for $M = 8$ and highest for $M = 512$. The trends are in line with expectations [47]. For M-PSK, from Figure 4.14, as the order of modulation M increases, the BER also increases which is expected.

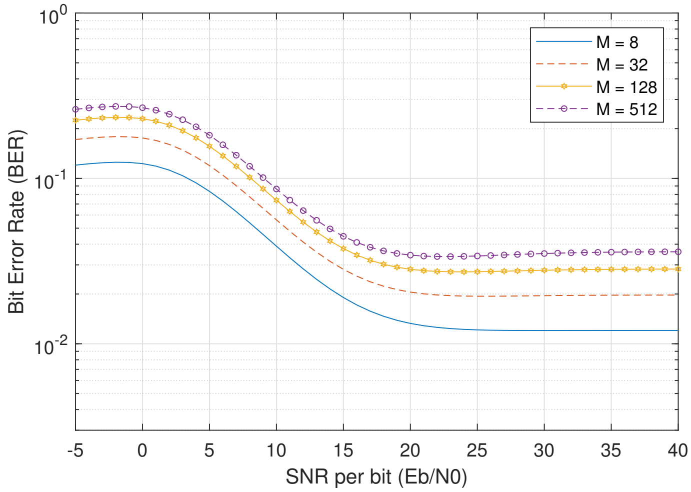

Figure 4.13: BER of Non-Rectangular M-QAM for collision case.

BER is lowest when $M = 4$ and is highest when $M = 32$. Also, as the SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations. The BER curve of M-PSK in Figure 4.14 with $M = 2$ and $M = 4$ exactly match each other. This is expected because the BER performance of BPSK (M=2) nd QPSK (M=4) are the same [32, 47].

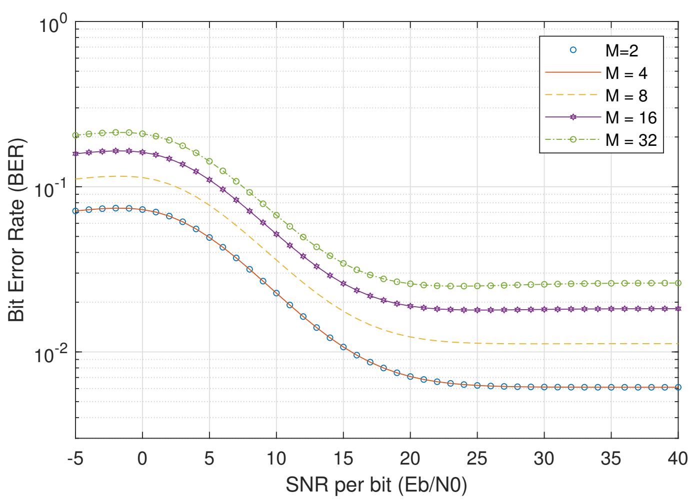

Figure 4.14: BER of M-PSK for collision case.

A plot for M-PAM (M = 2,4,8,16 and 32) is shown in Figure 4.15. As the order of modulation M increases, the BER also increases which is in line with expected values. Also, as the SNR is increased, the BER decreases which matches perfectly with expectations [32].

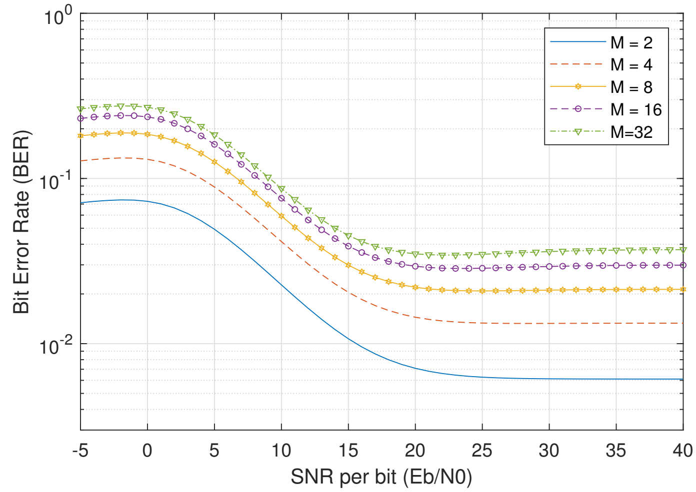

Figure 4.15: BER of M-PAM for collision case.

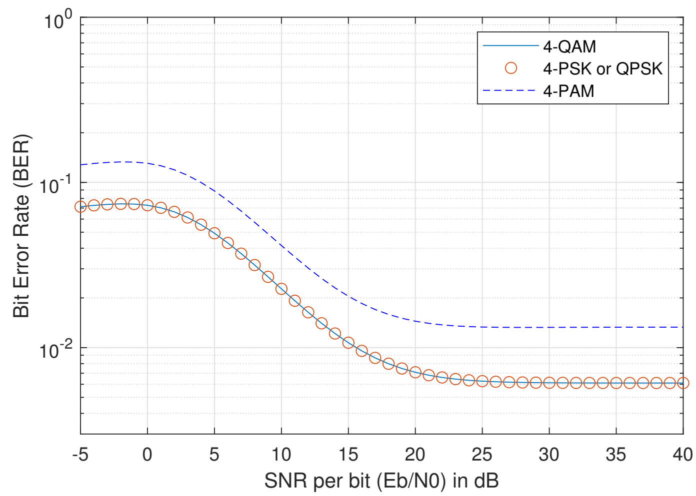

Figure 4.16: Comparison of BER for different Modulation schemes with M=4.

A plot QAM, PSK and PAM with M=4 is shown in Figure 4.16. The trend is in line with expectations perfectly as 4-QAM and 4-PSK are on top of each other. It is also expected that PAM will have a higher BER than QAM and QPSK. So the trend of the curves in the plot make perfect sense. The above plots were obtained for the collision or interference case using 4.23. In this case, both PU and SU are competing for transmission rights and SU might interfere with PU data causing undesired effects at the receiver. As we can see, the plots for collision and no-collision case are almost similar, except that no collision case has a higher BER, which is expected. In (4.21), η is 1, P_s is varied between $-30dB$ to $40dB$ and ψ is considered as $5dB$ while PU power P_p is assumed to be $25dB$. The above plots stick to the desired trend in that, as the SNR improves, the BER decreases. The BER first starts to decrease gradually and then saturates, meaning that an increase in SU power will not effect system BER. This is due to power adaptation because power adaptation makes sure the power of the SU does not increase beyond a certain limit. The IT constraint or ψ limits the increase of SU power and hence, the BER curve saturates. It can also be seen that for all plots, the BER curves saturate after $17dB$. This makes sense because the IT or ψ was considered as $17dB$ for the above cases.

4.6.3 Plots for Mean BER

The mean BER is obtained using equation (4.6).

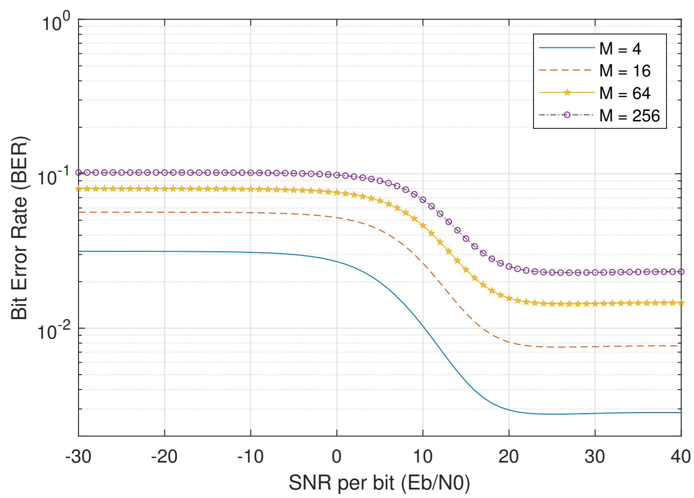

Figure 4.17: Mean BER of Rectangular M-QAM.

The plot for Rectangular M-QAM (M = 4, 16, 64 and 256) is shown in Figure 4.18. As the order of modulation M increases, the BER also increases which is expected.

BER is lowest when $M = 4$ and is highest when $M = 256$. Also, as the SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations. The same can also be seen for non-Rectangular M-QAM (M = 8, 32, 128 and 512) as in Figure 4.13, here, BER is lowest for $M = 8$ and highest for $M = 512$. The trends are in line with expectations [47, 32].

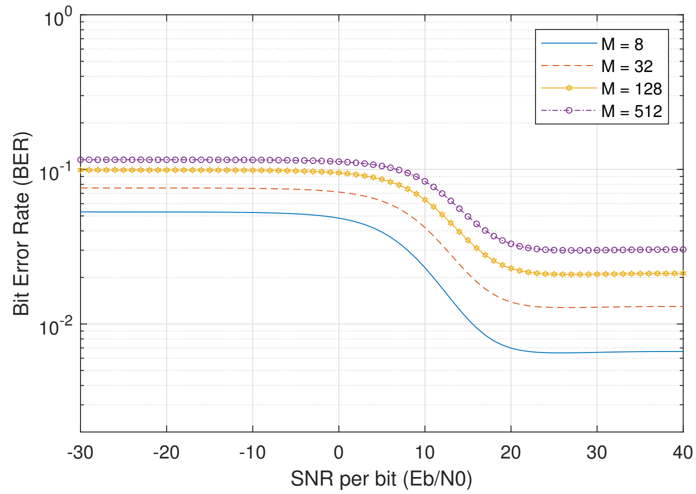

Figure 4.18: Mean BER of Non-Rectangular M-QAM.

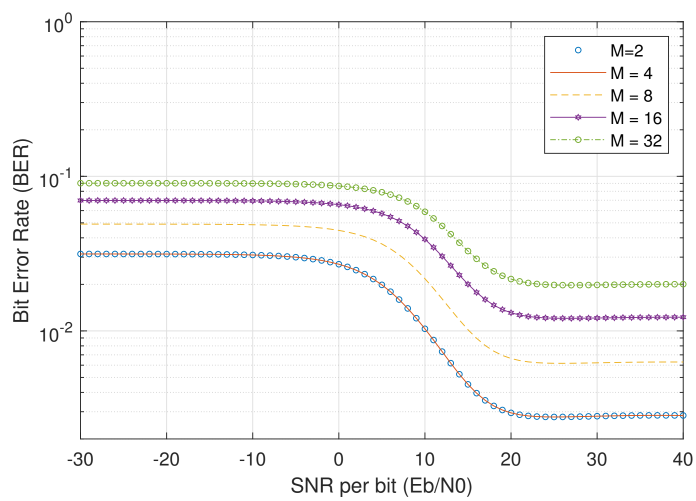

Figure 4.19: Mean BER of M-PSK for collision and no collision case.

For M-PSK, as the order of modulation M increases, the BER also increases which is expected. BER is lowest when $M = 4$ and is highest when $M = 32$. Also, as the

SNR is increased, the general trend of BER decreases gradually until it saturates. This matches perfectly with expectations. The BER curve of M-PSK in Figure 4.19 with $M = 2$ and $M = 4$ exactly match each other. This is expected because the BER performance of BPSK (M=2) and QPSK (M=4) are the same [32, 47].

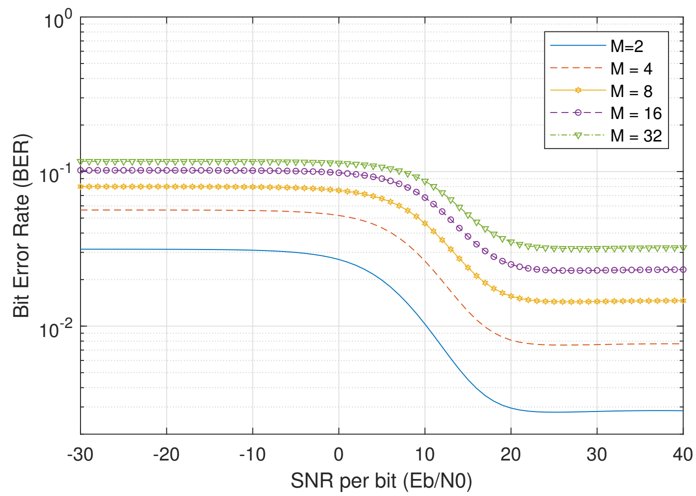

Figure 4.20: Mean BER of M-PSK for collision and no collision case.

A plot for M-PAM (M = 2, 4, 8, 16 and 32) is shown in Figure 4.20. As the order of modulation M increases, the BER also increases which is in line with expected values. Also, as the SNR is increased, the BER decreases which matches perfectly with expectations.

A plot QAM, PSK and PAM with M=4 is shown in Figure 4.21. The trend is in line with expectations perfectly as 4-QAM and 4-PSK are on top of each other. It is also expected that PAM will have a higher BER than QAM and QPSK. Therefore the trend of the curves in the plot make perfect sense.

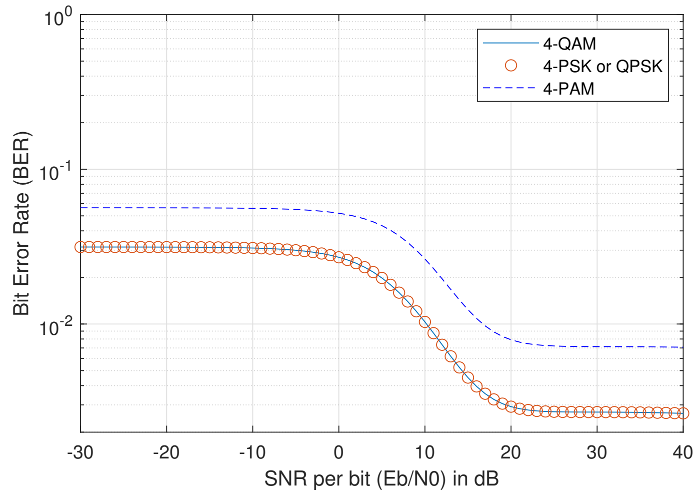

Figure 4.21: Comparison of BER for different Modulation schemes with M=4.

From (4.6), η is considered to be 2, P_s or SU transmit power is varied between $-30dB$ to $40dB$. ψ is considered as $17dB$ and the PU transmit power P_p is considered $50dB$. The total number of available carriers was considered to be 100, out of which 60 is reserved for the PU. 20 carriers are reserved for SU. 1000 iterations are considered. Substituting all the values in (4.6), we obtained the plots. The above plots stick to the desired trend in that, as the SNR improves, the BER decreases. This shows that the system performs better with high SNR as the number of errors are low. The BER remains constant for about -30 to $-10dB$ and the gradually decreases until almost $17 - 20dB$. The BER then saturates again and remains constant for the rest of the SNR, meaning that even if the power of SU is increased further, the BER does not improve. This is due to power adaptation because power adaptation makes sure the power of the SU does not increase beyond a certain limit. The IT constraint limits the increase of SU power and hence, the BER curve saturates.This makes sense because the IT constraint is $17dB$. If the power of the SU is increased more than the IT, the CR system will not allow the SU to increase its power further, meaning performance will saturate for power higher than $17dB$ can also be seen that for all plots, the BER curves saturate after $17dB$. This makes sense because the IT was considered as $17dB$

for the above cases.

4.7 Relationship Between BER and Interference Temperature

The relationship between how BER varies with varying ψ values is investigated in this section. Assuming constant PU transmit power of $50dB$, SU transmit power is varied between $-30dB$ to $40dB$. Three values of IT or ψ are considered, namely $10dB$, $20dB$ and $30dB$. A BER plot is obtained only for the Rectangular M-QAM case for simplicity. It can be shown in Figure 4.22.

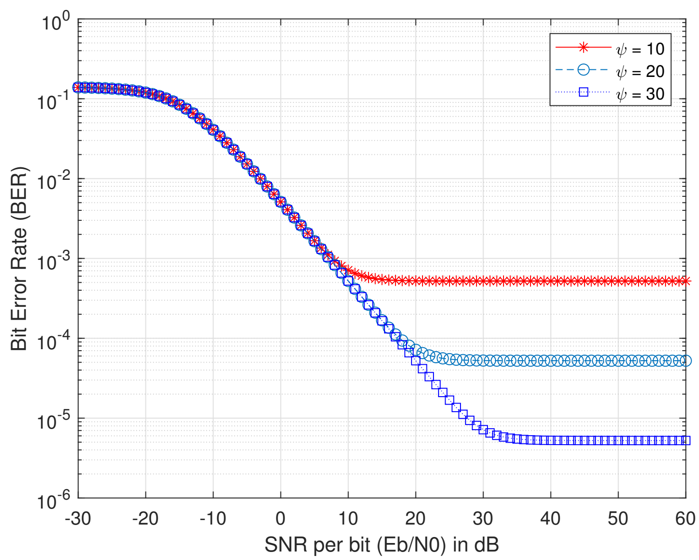

Figure 4.22: Comparison of BER for different values of interference temperature.

The results show that the BER curve saturates at $10dB$ for curve with ψ value of $10dB$. The BER curve saturates at approximately $20dB$ for curve with ψ value of $20dB$ and similarly, the BER curve saturates at $30dB$ for curve with ψ value of $30dB$. These results match perfectly with expected results. IT is a parameter which does not allow SU transmit power to increase beyond a certain level, to protect the QoS of PU and reduce interference. Thus, it makes sense that the curves start to saturate as they reach the IT constraint. The SU power cannot increase beyond this value, meaning the BER of the curve will not reflect any changes.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusions

In this thesis, in the first chapter, the spectrum crisis was briefly introduced. CR was presented as a a solution for this. In the second chapter, key terminology for CR was defined and a basic understanding of CR performance was presented. Chapter three dealt with BER and modulation schemes and how they are related to this thesis.

In the fourth chapter, BER performance of interference spreading in CR was investigated. Initially, assumptions for the mathematical analysis are presented. Then, a detailed analysis of the interference spreading concept is presented. Next, the system model is presented. The PDF for the collision and no collision cases. The mean BER is also derived. Finally, BER plots are obtained for the three different cases. Next, a brief section discussing the relationship between BER and IT is presented.

The main contribution of this thesis is the BER analysis of a CR employing interference spreading method. Literature suggests that most of the performance analysis for CR systems is done with the use of spectrum sensing method [9]. Spectrum sensing is the most commonly used interference management technique [9]. However, it has certain issues like high power consumption, unreliability in dynamic environments, high complexity etc. which are elaborated in this thesis [52]. To avoid the problems due to the use of spectrum sensing, a novel technique called interference spreading method is considered. Based on initial evaluations, interference spreading method is considerably reliable and power efficient compared to spectrum sensing. The BER plots and initial evaluations suggest that interference spreading method can be a highly reliable alternative to spectrum sensing. The plots obtained for various modulation schemes match initial expectations, further validating the use of interference spreading method as an alternative and complementary technique to spectrum sensing.

5.2 Future Work

Interference spreading as an interference management technique is a relatively new technique [9]. Most of the work done in CR is done with the use of spectrum sensing. Hence, the major contribution of this thesis was the performance analysis of interference spreading in CR. This thesis only presented initial evaluations and a lot of future work can be done. Closed-form equation can be obtained for the collision, no collision and mean BER cases. Next, the performance analysis can also be extended to consider the system capacity. As this thesis only considers a single PU and a single SU, the work done can be extended to the performance of a CR system with multiple users.

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